

MODELLING REPLACEMENT MIGRATION

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Complicated demographic situation in Russia and its regions is characterized by the negative rate of natural increase which is not compensated by positive net migration. Thus, the total population size decreases (see Fig. 1) leading to numerous socio-economic problems. Population reproduction is determined by natural movement and migration. Till the 1990th the natural increase in Russia was positive and contribution of net migration to population growth was not significant (see Fig. 2a,b). But when the natural increase is negative, fertility is much lower than the replacement level and its significant increase in the future is not expected there is no source of population non-decrease different from immigration.

In the year 2000 the UN concept of replacement migration, i.e. migration that compensates negative natural increase or population ageing, appeared. In the mentioned publication as well as in many empirical studies population projections based on different scenarios were considered. The paper aims at applying an extended Leslie matrix model to study replacement migration for low fertility populations.

At first the Leslie matrix model with migration is considered then the corresponding computer-based technique of modelling replacement migration is applied to populations of Russia and its greatest non-metropolitan megacity Saint-Petersburg.

For computations Mathcad 2001 Professional has been used. Vital statistics given by Goskomstat of the Russian Federation, Peterburgkomstat, Centre D'Estudis Demographics Universitat Autònoma de Barcelona is used.

In order to simplify the problem at the first stage of our study it is assumed that the reproduction regime remains constant over a period of time, and the female population of reproductive ages alone is considered.

If column-vector $\mathbf{n}(\mathbf{t}) = (n_0(t), n_1(t), \dots, n_{\beta-1}(t), n_{\beta}(t))$ represents female population at time t , $n_i(t)$, $i = 0, 1, \dots, \beta$, - the number of females alive in 5-year age group i , β - the last age group within which reproduction occurs (here $\beta=10$), then

$$\mathbf{n}(\mathbf{t}_1) = \mathbf{L}\mathbf{n}(\mathbf{t}), \quad t_1=t+5, \quad (1)$$

where \mathbf{L} is the $(\beta+1)(\beta+1)$ Leslie matrix. Mathematical properties of the Leslie matrix are well studied: being non-negative and indecomposable, \mathbf{L} has a positive eigenvalue λ_0 of multiplicity 1 and moduli of all other eigenvalues are smaller than λ_0 , to λ_0 corresponds a positive eigenvalue \mathbf{v}_0 and

$$\lim_{t \rightarrow \infty} \frac{\mathbf{n}(t)}{\lambda_0^t} = \pi \mathbf{v}_0 \quad (2)$$

(π is some constant).

Expression (1) describes reproduction of closed populations. Consider now open populations, only net migration being considered. In this case the process of reproduction may be described by the equation

$$\mathbf{n}(\mathbf{t}_1) = \mathbf{L}\mathbf{n}(\mathbf{t}) + \mathbf{R}\mathbf{n}(\mathbf{t}) \equiv \mathbf{L}_m \mathbf{n}(\mathbf{t}), \quad (3)$$

where $\mathbf{R} = \text{diag}(\mathbf{r})$, $\mathbf{r} = (r_0, r_1, \dots, r_{\beta})$ - vector of age-specific net migration rates. \mathbf{L}_m is quasi-non-negative (all its non-diagonal elements are non-negative). For (3) an expression analogous to (2) takes place with μ_0 being the real eigenvalue of \mathbf{L}_m having the maximal real part. In fact for all populations considered the following inequalities take place

$$|\mu_i| < \mu_0, \quad \mu_i, \quad i = 1, 2, \dots, \beta, \quad (4)$$

avoiding periodicity. But in the computer program spectrum of \mathbf{L}_m is computed and inequalities (4) are checked.

For Russia/Saint-Petersburg population increase seems neither plausible nor desirable, thus it presents interest to find such \mathbf{r} which under a fixed reproduction regime ensures in the long run population size stability ($\mu_0 = 1$). This means finding \mathbf{r} from the characteristic equation for \mathbf{L}_m where μ_0 is assumed to be 1.

To avoid non-uniqueness it is supposed that components of \mathbf{r} should satisfy some additional reasonable equations and/or inequalities. First of all, as it follows from observed data $\mathbf{r} < \mathbf{1}$. Here the following types of age distributions of migration are considered:

- 1) “uniform” migration ($r_i = 1 - \lambda_0$, $i=0, 1, \dots, \beta$) – denoted **uni**;
- 2) “observed” distribution (\mathbf{r} is computed based on an additional vector \mathbf{a} reflecting a real (observed) migration structure – denoted **obs**;
- 2a) “youth” migration (for ages under 35 or 40 components of \mathbf{r} in 2a) coincide with case 2), for older ages they are almost zero) – denoted **you**;
- 3) \mathbf{r} is computed based on the UN model pattern - denoted **UN**.

As an example dynamics of female populations of Russia (base year 1997)/Saint-Petersburg (base year 1999) in the long run are considered under different types of migration structures. Main demographic indicators for Russia 1997/ Saint-Petersburg 1999 are given in Tab. 1, 2. As examples of real migration profiles, which are used for finding \mathbf{r} , those for Russia, Saint-Petersburg, Finland and Spain for different years have been taken, i.e. for Russia 1997, Saint-Petersburg 1989 and 1999, Finland 2001 and Spain 1999. Real migration distributions may vary within a very wide range, e.g. some components may be negative, big differences in sizes of adjacent age groups may take place etc.

All considered scenarios assume constant reproduction regime (Saint-Petersburg 1999 and Russia 1997) and migration structures based on the following distributions: : uniSPb99, obsSPb89, obsSPb99, obsRu97, obsSp99, obsFin2001, youRu97, UN (for Saint-Petersburg), uniRu97, obsSPb89, obsSPb99, obsRu97, obsSp99, obsFin2001, youRu97, UN (for Russia). Computed age distributions of migration for Saint-Petersburg/ Russia for scenarios uniSPb99/uniRu97, obsRu97, obsSp99, youRu97, UN are given on Fig. 3, 3R. Migration distributions both for Saint-Petersburg and Russia based on obsSPb89, obsSPb99, obsFin2001 stand out for negative components and wide range of age group sizes (see Fig. 4, 4R).

Migration age structure affects the size of migration stream and thus the total size of the limit population. Fig. 5, 5R represent the total population size dynamics of female populations of Saint-Petersburg/ Russia in reproductive ages according to the given scenarios and with zero migration (CR-SPb99, CR-RU97).

In Tab. 3, 4 results of computation of annual net migration for considered scenarios and changes in the total population size relative to the base year are given. It can be seen that more "regular" migration distributions both for Russia and Saint-Petersburg result in smaller migration streams, while quite close distributions uniSPb99 and obsRu97 (uniRu97 and obsRu97) lead to maximal ones. Of course, these scenarios with greatest migration streams correspond to the greatest values of population size.

Age distributions of limit populations are given on Fig. 4, 4R, 6, 6R.

The described technique allows assessment of migration streams that could ensure stable population dynamics. For each considered variant of migration the corresponding limit population is computed. Besides, for a fixed \mathbf{r} it is possible to find such fertility rates that would provide an asymptotic stationary state.

Results of the study may be used when elaborating migration policies at the country and/or regional level.

Table 1. **Main demographic indicators, Saint-Petersburg, 1999**

Total size	(thousand)	4660.8
	males	2101.7
	females	2559.1
TFR		0.90
LE	males	61.6
	females	73.1
Net migration	(thousand)	9.3
	males	2.1
	females	7.2
	λ_0	0.85

Table 2. **Main demographic indicators, Russia, 1997**

Total size	(thousand)	
	males	68926.3
	females	78012.0
TFR		1.23
LE	males	60.9
	females	72.8
Net migration	(thousand)	352.6
	λ_0	0.90

Table 3. Annual net migration for scenarios uniSPb99, obsSPb89, obsSPb99, obsRu97, obsSp99, obsFin2001, youRu97, UN and changes in the total population size relative to the base year 1999 for Saint–Petersburg 1999

№	Scenarios for migration	Annual net migration (thousand)	population size in 2099 / population size in 1999
1	uni SPb99	52.5	1.32
2	obs SPb99	21.9	1.08
3	obs SPb89	19.6	1.03
4	obs Ru97	52.4	1.34
5	obs Sp99	44.0	1.26
6	obs Fin2001	17.5	1.20
7	you Ru97	36.1	1.21
8	UN	42.0	1.30

Table 4. Annual net migration for scenarios uniSPb99, obsSPb89, obsSPb99, obsRu97, obsSp99, obsFin2001, youRu97, UN and changes in the total population size relative to the base year 1997 for Russia 1997

№	Scenarios for migration	Annual net migration (thousand)	population size in 2097 / population size in 1997
1	uni RU97	1168.7	1.36
2	obs SPb99	563.6	1.20
3	obs SPb89	507.6	1.16
4	obs Ru97	1173.1	1.39
5	obs Sp99	1034.9	1.32
6	obs Fin2001	428.6	1.30
7	you Ru97	828.5	1.29
8	UN	1005.8	1.36

Fig.1. Total population size dynamics, Russia and Saint-Petersburg, 1959-2002 (mln.)

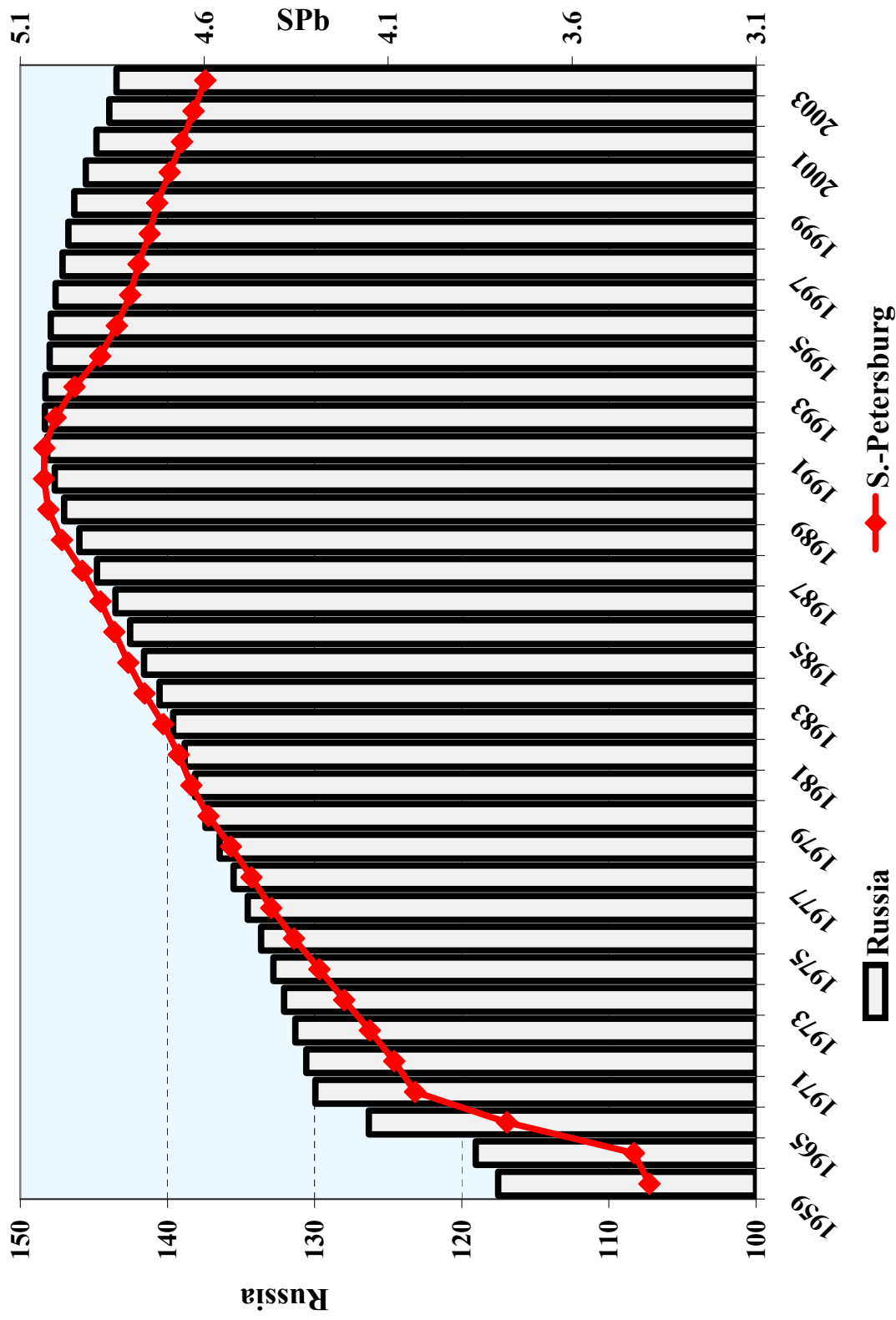


Fig.2a. Components of population size changes, Russia, 1959-2001 (thousand)

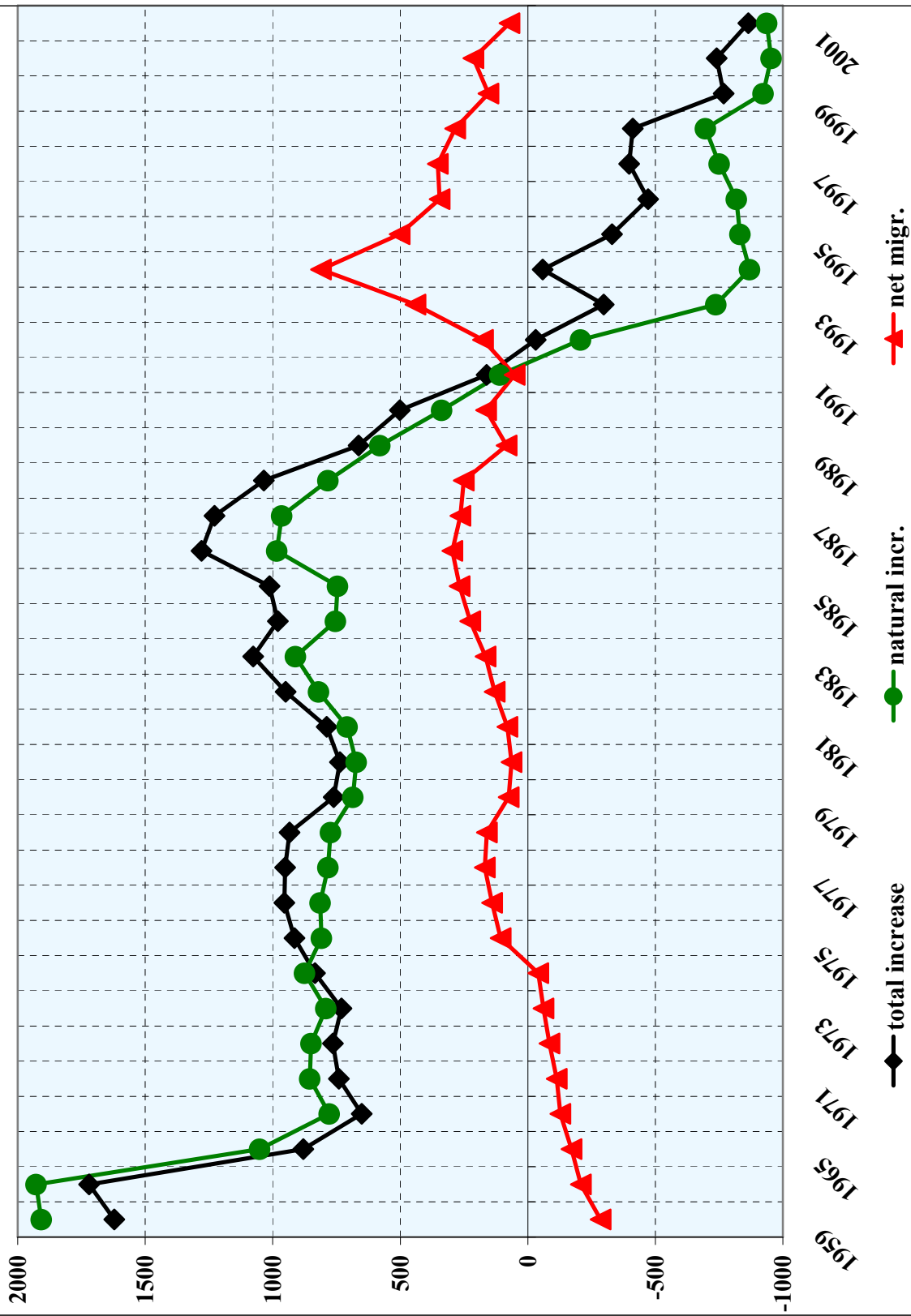


Fig.2b. Components of population size changes, Saint-Petersburg, 1959-2002 (thousand)

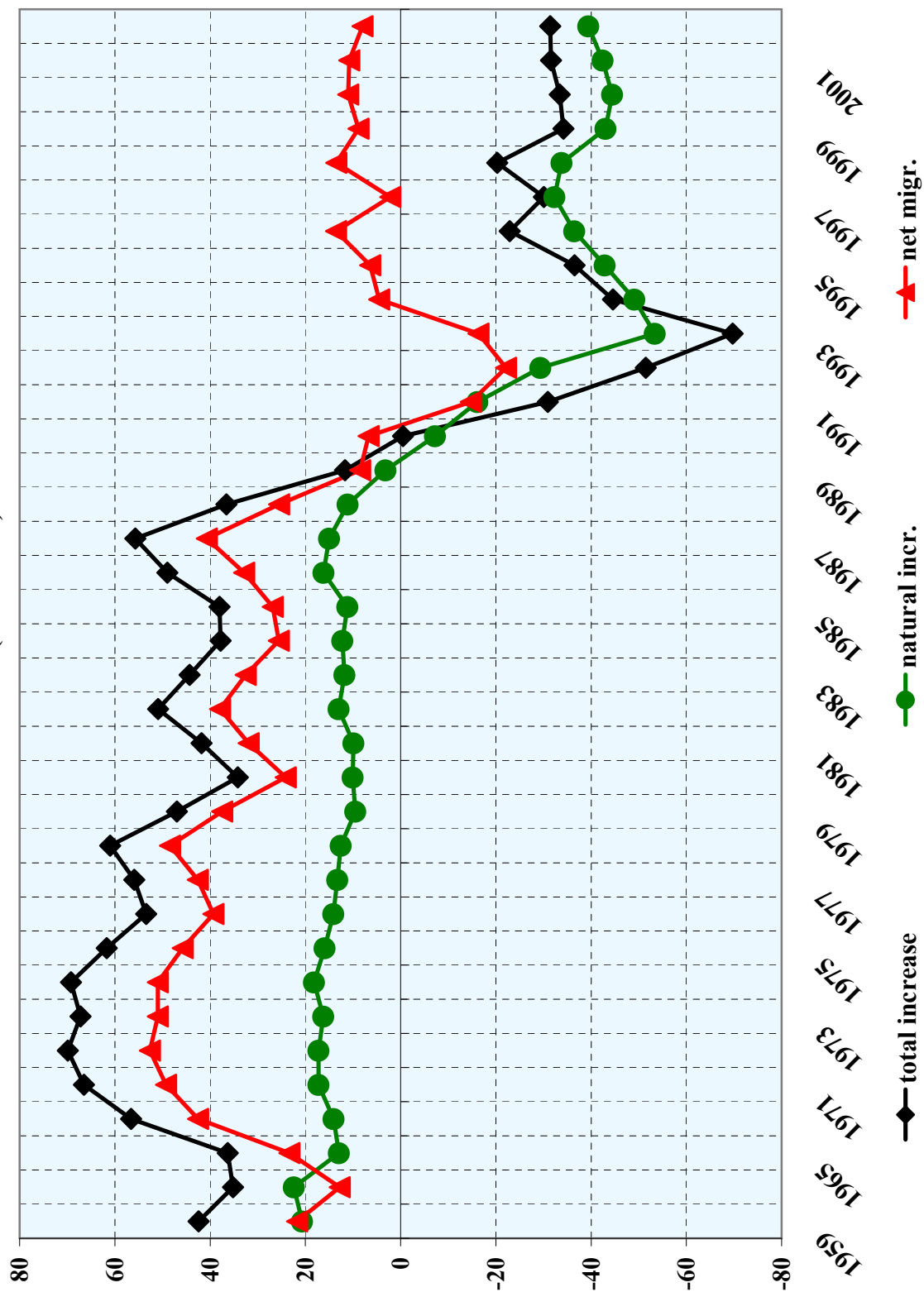


Fig.3. Age distributions of net migration for scenarios uniSPb99, obsRu97, youRu97, obsSP99, UN (%)

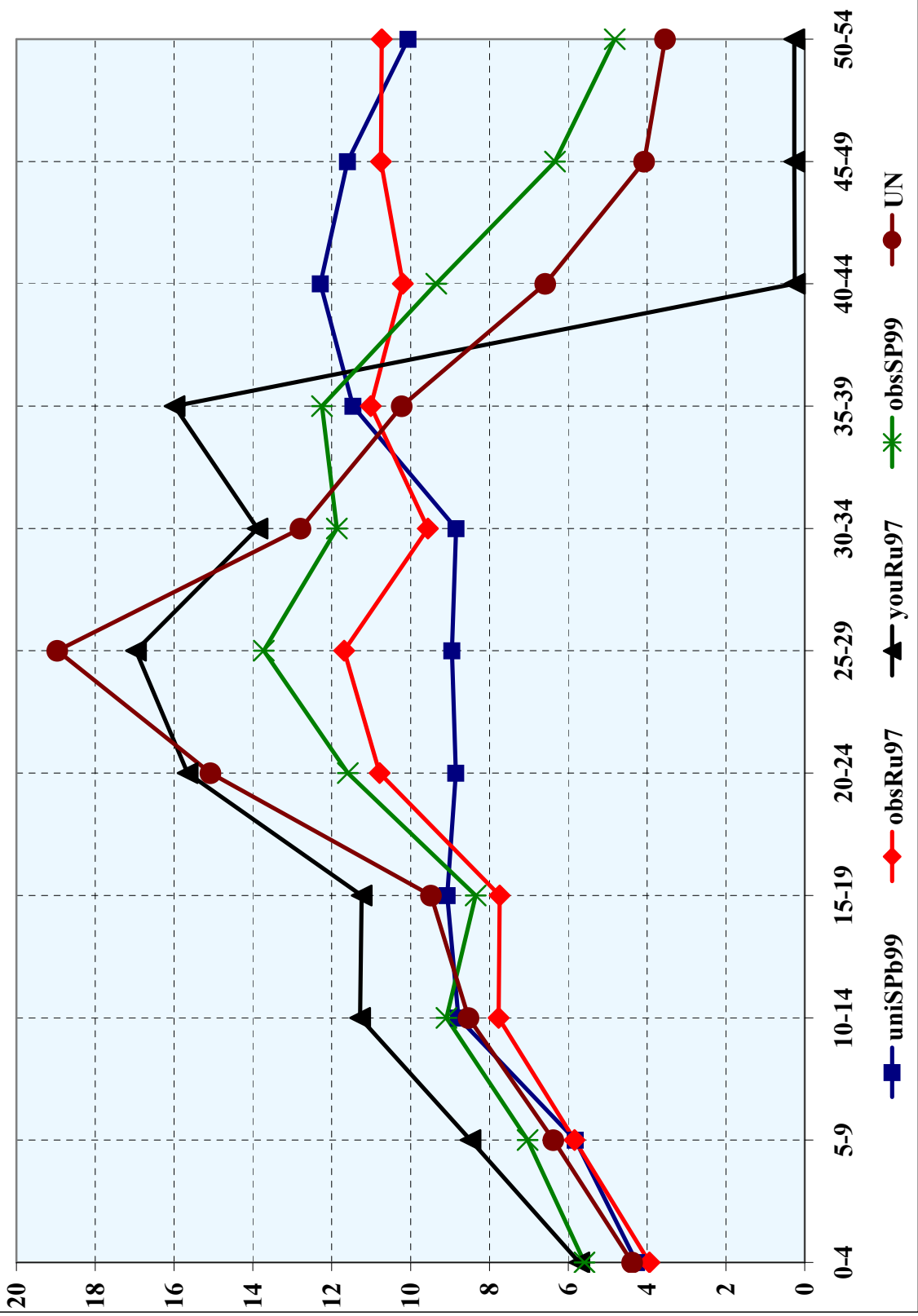


Fig. 3R. Age distributions of net migration for scenarios
uniRU97, obsRu97, youRu97, obsSP99, UN (%)

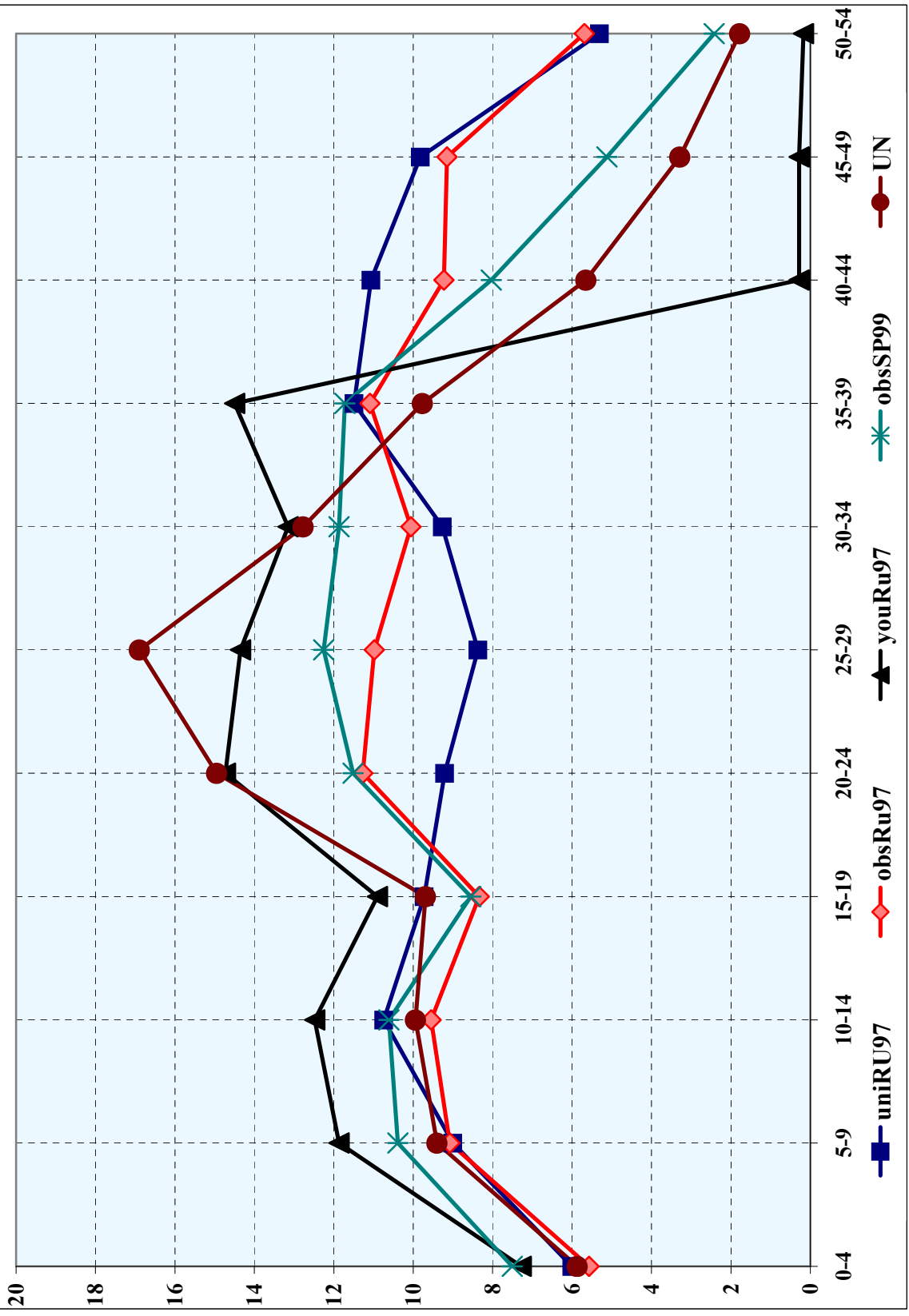


Fig 4R . Age structure of limit populations (%) and age distributions of net migration (%) for scenarios obsSPb99, obsSPb89, obsFin2001

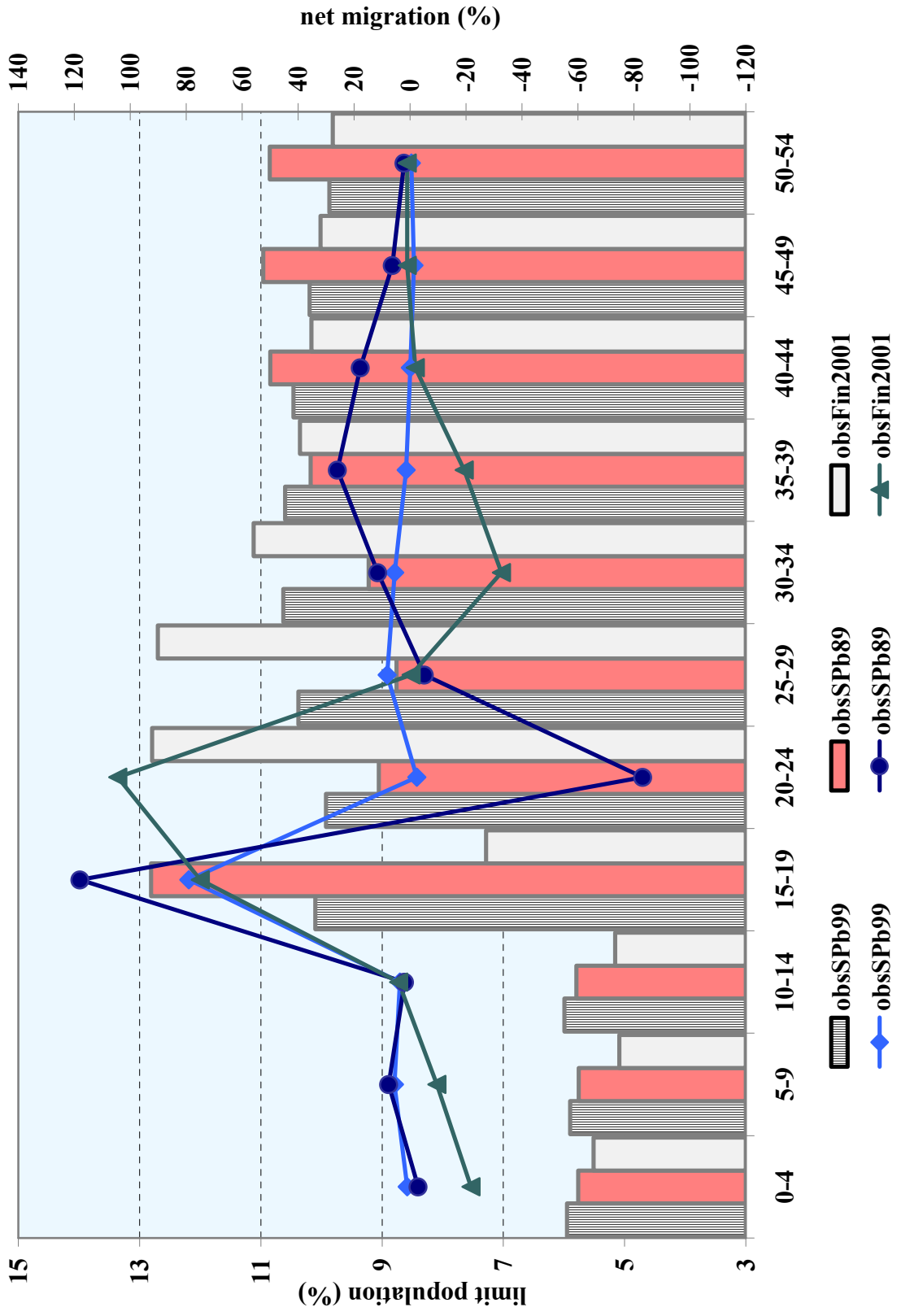
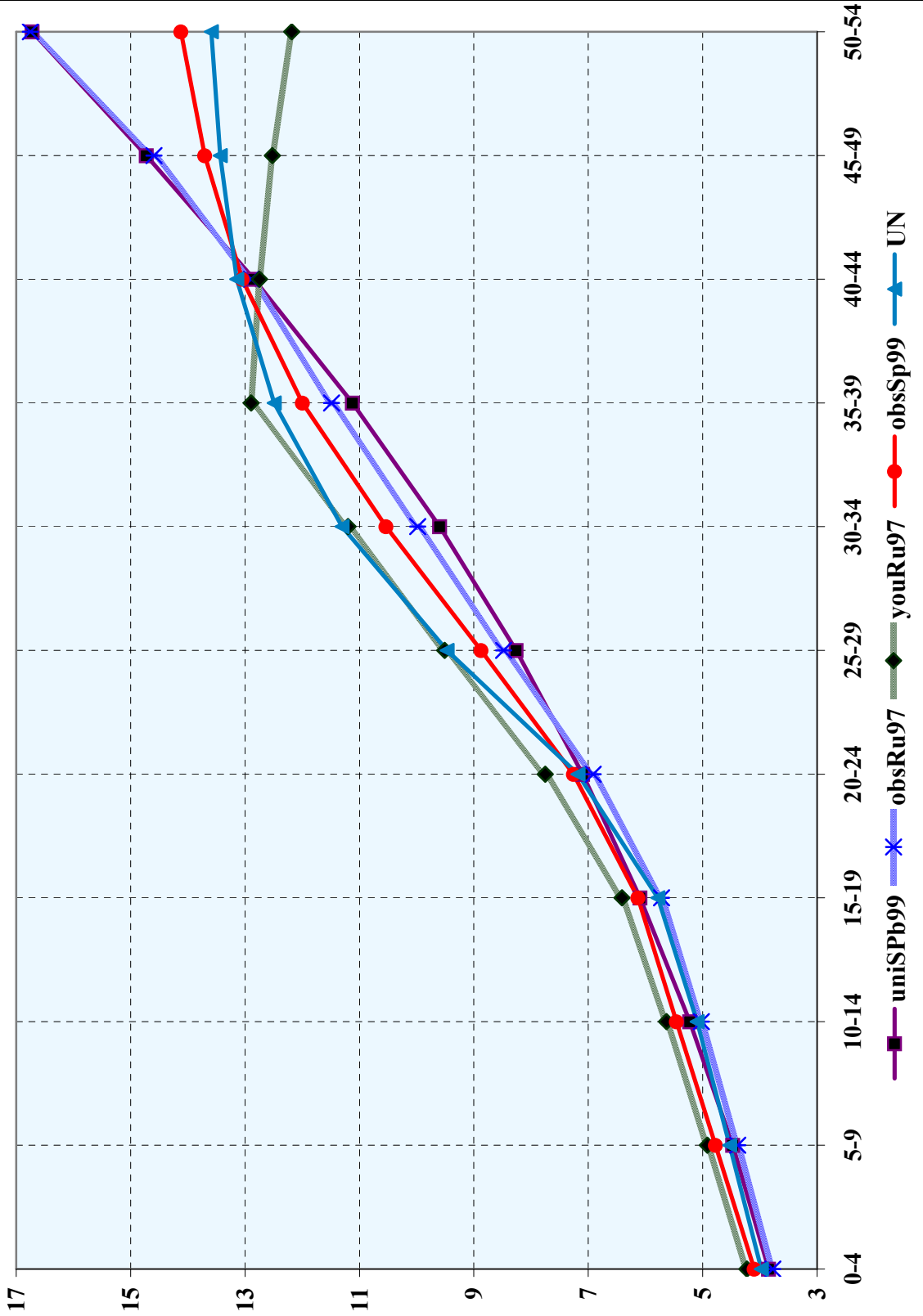


Fig.6. Age structure of limit populations (%) for scenarios uniSPb99, obsRu97, youRu97, obsSp99, UN



**Fig. 6R . Age structure of limit populations (%) for scenarios
uniSPb99, obsRu97, youRu97, obsSp99, UN**

