[Version: June 3, 2005]

A Stochastic Simulation of the German Population in 2050

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Abstract

This paper presents a stochastic projection of the German population, using recently developed models. This is not straightforward, since reunification in 1990 creates a structural break in the East German time series. A common and consistent time series for West and East Germany as a whole does therefore not exist. It is shown that since unification East German population parameters have converged to West German levels and that this adaptation process of considered to be almost completed by now. Consequently, German population parameters can be modelled using only West German historic patterns, whereas the start-off population is of entire Germany.

1. Motivation

Official projections rest on deterministic models, which do not adequately reflect the uncertainty of future demographic rates[1]. Based on certain combinations of demographic rates, population developments are calculated. This method has been employed in official projections for decades though it has mainly two deficiencies: First, it cannot provide information on the probability of a certain scenario. Second, modeling uncertainty by means of different scenarios is necessarily wrong, due to unrealistic assumptions on the correlation structure of forecast errors[2].

To take account of these problems, probabilistic approaches to population forecasting have been developed. The principal objective of these approaches is to obtain prediction intervals of demographic variables and thus to measure forecast uncertainty. In stochastic population projections, forecast errors are propagated over time. Depending on correlation patterns, forecast errors either reinforce or cancel out each other over time. The stronger the autocorrelation of the rates, the weaker is the tendency of the forecast errors to average out. This is why a correct specification is crucial. For projections on the national level four types of correlations are of importance (Keilman et al. 2002), temporal correlation or autocorrelation of demographic rates, correlation between rates, correlation between age groups, and finally correlation between sexes.

The autocorrelation of fertility is usually high. One period with high fertility is typically followed by another period with a high fertility. As migration depends strongly on the political and economical environment, which changes faster than patterns of reproduction, serial correlation should be weaker for migration than for fertility. Similarly, the correlation pattern of mortality is a priori unclear. Regarding correlation between the rates, there is no reason to assume that mortality, fertility and migration are strongly correlated in developed countries (Lee 1998). Concerning the correlation of rates of adjacent age groups, one usually assumes a positive correlation. This also applies to correlation between the sexes. Here also forecast errors do not compensate but reinforce each other.

Currently, there are three different approaches to stochastic population forecasting. The first approach rests on the analysis of historical forecast errors (Keyfitz 1981, Stoto 1983). By comparing former projections with observed population developments, one calculates the standard error of forecast, which is used to construct the forecast interval. Keilman et al. (2002) point out, that only short time series of historical forecast errors are available and that forecast errors may have reduced by better methods and more experience.

A second approach uses experts' assessments on both future trends of demographic rates and on the degree of uncertainty of these quantities (Lutz, Sanderson, and Scherbov 1996). Lutz and Scherbov (1998) have developed a stochastic population projection based on this method. Assuming the distribution of the rates and taking assumptions about the serial correlation of different rates, a probability distribution is simulated. Lee (1999) argues, that even experts are hardly able to distinguish between a 95% and a 99% prediction interval.

The third approach was pioneered by Ronald Lee (Lee and Carter, 1992; Lee, 1993; Lee and Tuljapurkar, 1994), who uses time series methods to project population parameters, such as

[1] Given the population in year t-1 is correct, the forecast uncertainty reduces to uncertainty of the rates in the Leslie-matrix, and the migration.

[2] A detailed methodological discussion can be found in Lee (1998).

fertility and mortality rates. Probability distributions of the respective parameters are then generated by means of a stochastic simulation. Only time series models allow for a consistent consideration of forecast uncertainty and the article at hand is essentially based on this method.

The paper is organized as follows. Section 2 shortly describes the data, in section 3 we model fertility using a quadratic spline approach. In section 4 we model mortality using the well known approach by Lee and Carter. Section 5 briefly reviews the migration model while section 6 discusses main results of the relevant population projection parameters. Section 7 concludes.

2. Data

The mortality data used in this study were obtained from the Human Mortality Database[3]. This database contains annual age- and sex-specific mortality data from 1956 to 1999. The time series of population rates required were collected from the Human Mortality Database Data for higher age groups, the first and latest years as well as age specific fertility data stem from Statistics Germany[4]. All rates are derived from absolute quantities i.e. only absolute birth numbers are used. Using absolute quantities results in an easier plausibilisation of not consistent rates.

Separate data for West and East Germany is available only until 2000. Decomposing data of unified Germany into East and West turned out to be not viable. As start-off population serves the German population on 31.12.2002.

3. Fertility

3.1 Modelling Age Specific Fertility Rates

Since the 50s of the 20th century, structural breaks have characterised fertility in West Germany. The interval from 1954 to 1966 is e.g. characterised by high fertility rates, culminating in 2.54 births per woman in 1964. After 1966 fertility rates declined sharply. Since 1973, the West TFR is relatively stable at around 1.4 children per woman. However, fertility behaviour still changes: the upward trend of the mean age at childbearing is still ongoing. The mean age of mothers at childbearing has risen from 26.8 to 28.9 years in West Germany since 1973. However, in 1950 it was almost as high as today (28.6 years). This trend has led to increasingly symmetric age-specific fertility rate schedules.
In order to parameterise the age-specific fertility rates (ASFR), the Coale and Trussell (1974) model seems to be the most frequently applied model. It uses three shape parameters and a level parameter (TFR) within a double exponential function. However, for contemporary West European countries, it does not fit very well (Schmertmann 2003). Thompson et al. (1989) fit ASFR using a Gamma density curve. The Gamma density has three parameters and

[3] www.mortality.org.
[4] Statistisches Bundesamt, www.destatis.de.

performed well especially in the early 1980s. Due to increasingly symmetric fertility curves, recent fertility patterns are fit rather poorly.
In Lipps and Betz (2004), the ASFR was parameterised with a Gaussian (bell shaped) curve, between 1973 and 2000. The three parameters μ (mean age of mothers at childbearing), σ (standard deviation of mean age of mothers at childbearing), and TFR were modelled separately:

1. μ was fitted by a logistic growth curve, resulting in the three parameters saturation level, expansion parameter (multiplier of $t-t_0$), and inflection point.
2. σ was fitted by a vector autoregression model using σ, TFR, and μ with one lag.
3. TFR was modelled as a random walk time series.

The bell-shaped curve has two advantages: First, the parameters have very easy interpretations and are easy to extrapolate, using not only pure time series models, but also suitable (albeit weak) assumptions about future developments[5]. Second, the approximation improves over time due to increasingly symmetric ASFR schedules. On the other hand, this procedure has two major drawbacks:

1. it does not take into account the cross-correlation of the three parameters.
2. it assumes symmetry throughout the forecast interval. Since structural stability is hypothesised, due to the increasing age of mothers at childbirth, the skewness of the curve might move further to the right in the future. Ever more non-symmetric ASFR schedules will however be interpolated increasingly worse by bell-shaped curves.

Schmertmann (2003) suggests a system of quadratic spline (QS) functions with four parameters. An appealing property of the QS approach is that it is very flexible and therefore can fit virtually any ASFR curve. However, when fitting recent western ASFR, the QS model cannot capture a small "hump" for young women around the age of 21. This deficiency however applies to all models discussed so far (Figure 1):

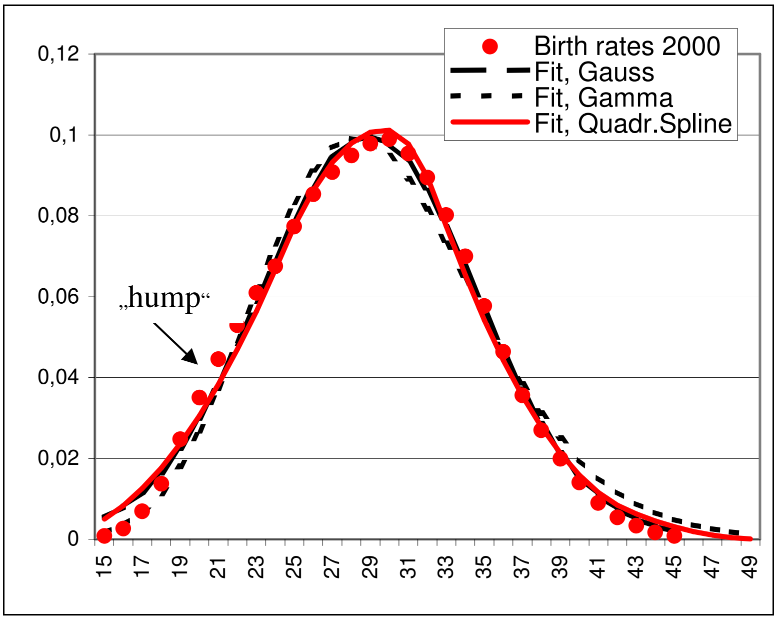

Figure 1: ASFR and different fitting curves, West Germany, 2000

[5] E.g. the logistic growth curve reflects the probably continuing postponement of childbirth on one hand, and its biological limits on the other.

The explanation for this failure and our suggestion to alleviate this problem requires some explanation about the mechanism of the QS approach[6]: The QS model piecewise concatenates quadratic polynomials in a smooth way, i.e. including the first derivative. The QS has five sampling points $t_0, \ldots, t_4$, which can be calculated as a combination of the following characteristic sampling points:

- α, the youngest age at which fertility rises above zero,
- P, the age at which fertility reaches its peak level ("modal" age in the following),
- H, the youngest age above P at which fertility falls to half of its peak level.
- β, the upper age limit (=49 fixed).

The second knot t_1 is defined by means of a parameter $W = \min [.75, .25 + .025(P-\alpha)]$. The first three sampling points are defined as follows:

- $t_0 = \alpha$
- $t_1 = (1-W)\ \alpha + WP$
- $t_2 = P$

The QS cannot capture the ASFR of younger women (see Figure) simply due to the lack of two sampling points between t_0 and t_2. That is, having only three sampling points available is not enough to describe the curve between t_0 and t_2, which has three turning points.

Fitting historic fertility rates as proposed by Schmertmann (2003) and extrapolating the sampling points into the future leads to implausibly low estimated starting ages of fertility after some years. Because NLS minimizes the quadratic distance between the spline function and the ASFR, fertility at very young ages is systematically overestimated. Using the model for stochastic projection purposes further aggravates the problem.

We therefore decided to fix α at age fifteen. Since our data also starts not until age 15, this is reasonable. The model then depends only on two parameters P and H. Of course, it is not able to fit the data as good as before. We improve the two-parameter fit by minimizing also over W. The fit of the ASFR in 2000 is depicted in Figure 2[7].

[6] For further details of the QS method confer Schmertmann (2003).
[7] SSE=9.11 * 10^{-5} in the modified (w-min) QS model vs. SSE=3.43 * 10^{-4} in the original QS model.

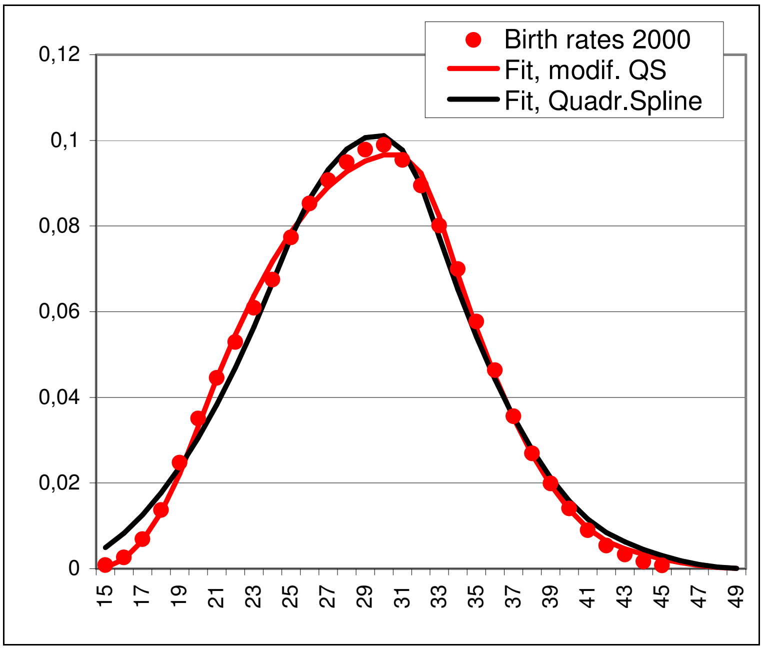

Figure 2: ASFR original, and modified QS fit, West Germany, 2000

In contrast to previous work (Lipps and Betz 2004), where we started to model ASFR in 1973, we keep the entire West German fertility time series since 1950. This reflects the uncertainty of a longer time span.

In order to model stochastic forecast trajectories, we conduct a Vector Autoregression (VAR) with the parameters[8] P, H and W, with two lags. The eigenvalue stability criterion is satisfied; all eigenvalues lie strictly inside the unit circle. This implies covariance stationarity[9] of the time series, i.e. the first two moments of the process are independent of t (Hamilton 1994, 46), confirming our intention to model without trend (Lee 2004). Tests on normality of the disturbances are however rejected, and order two lagged disturbances exhibit weak autocorrelations. However, a Wald test of the hypotheses that the variables at lag two are jointly zero is clearly rejected. As a consequence of these analyses, all coefficients of lag one and two are kept in the VAR equations. The historical development of P, H and W together with their forecasts and the upper and lower σ-bounds are depicted in Figure 3.

The VAR, however, has limitations. Parameter estimates are quite imprecise, that is standard errors of the estimates are fairly large. For instance, the projected standard error of P in 2050 equals 3.5 years. Hence, when performing stochastic simulations, some iteration results may yield a P greater than H. In this case the QS-model is ill-defined, because it requires H to be greater than P. In order to make sure that the model is well-defined, we restrict the simulated Ps to the interval between 20 and 40 years (compare Keilman et al., 2002, 421), and the simulated Hs to the interval between P+2 and 49 years[10].

[8] Actually we model the natural logarithm of the parameters, i.e. ln(p), ln(h), and ln(w).
[9] This holds although tests on stationarity of the univariate time series ln(p), ln(h), and ln(w) are rejected.
[10] We imposed the following restrictions on H: In case H<P+2, we define H=(H+P+2)/2, with the right hand side taken from the previous iteration. Similarly if H>49, we define H=(H+49)/2 (previous iteration).

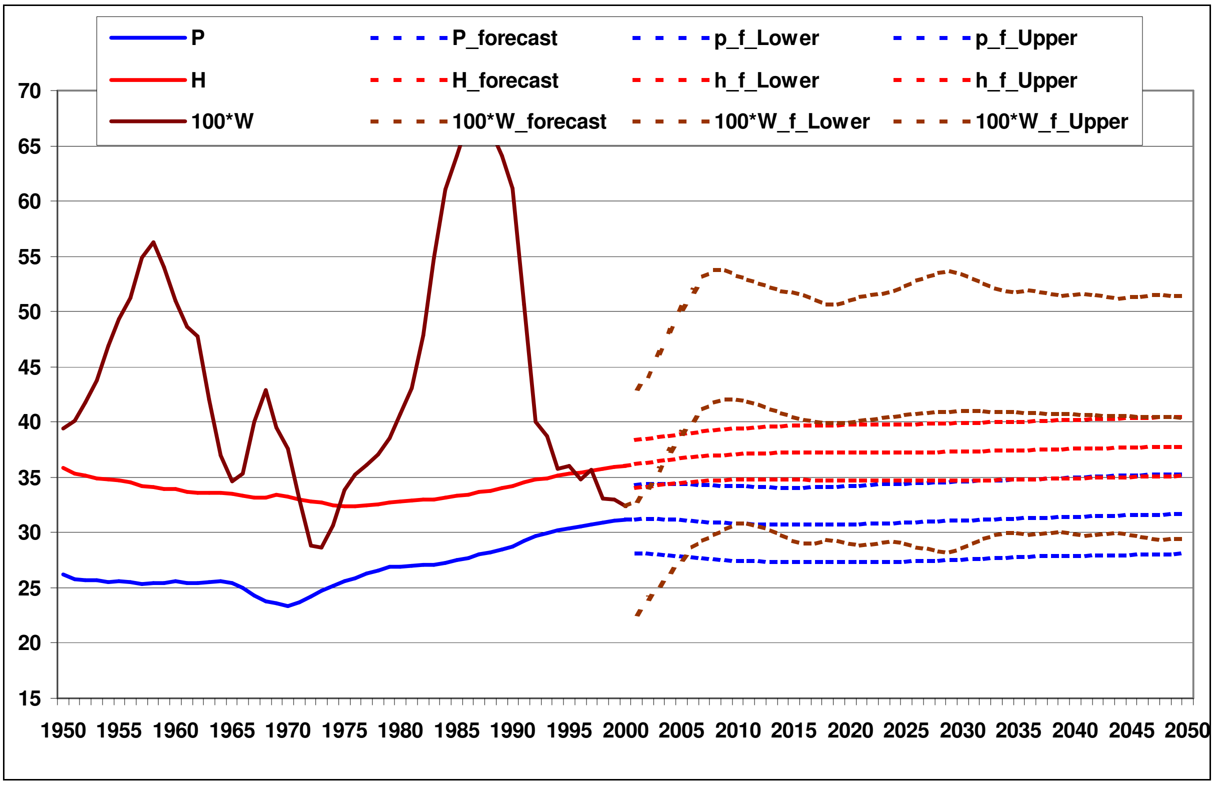

Figure 3: Results of the VAR, with simulated expected values and σ-intervals. West Germany

Due to stationarity of the VAR forecast, the prediction interval does not adequately take into account the increasing uncertainty with increasing forecast time. We modify the simulated trajectories by letting each trajectory start at the fitted value in 2000. The cross sectional prediction intervals of the simulated modal ages P are depicted in Figure 4.

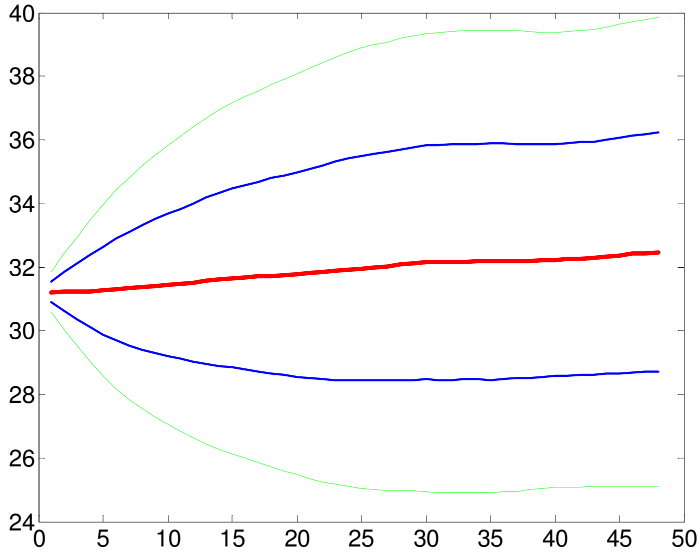

Figure 4: VAR-simulated modal ages of mothers at childbirth (P) with restrictions, corrected for future uncertainty. Forecast 2001-2050, mean (red) , +/- one (blue), and two (green) σ-intervals, West Germany

3.2 Total Fertility Rate

We model also TFR under the assumption that no systematic trends or structural breaks occur(ed). Therefore we consider the time series only from 1973 on, ignoring the structural breaks, which happened before the beginning 70s (baby boom and baby bust). However we do not impose a mean constraint, as e.g. in Lee (1993).

Although an AR(1) model would also fit standard Box-Jenkins requirements, we decide to model the TFR time series as a random walk, because of its parsimonious parameterisation. In addition, sensitivity analysis show that both models produce similar results. The differenced series shows no autocorrelation.

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
μ	1.404743	.0109391	128.41	0.000	1.383302	1.426183
σ_ε	.057884	.007202	8.04	0.000	.043769	.072

Table 1: Estimated parameters and goodness of fit of random walk model for TFR, West Germany

The average TFR between 1973 and 2000 amounts to 1.405 which is close to the forecast of Statistics Germany with 1.4 (Statistisches Bundesamt 2003). The forecast interval increases with the square root of the forecast horizon, such that a σ-interval of [0.995,1.814] results in 2050.

3.3 East German Fertility

In the 1950s and 60s the West and East German total fertility rates were at roughly the same levels. Sinve the early 70s the total fertility rate in the East is higher than in the West, due to the East German pro-natalistic policies (Kreyenfeld 2001). After unification the East TFR dropped sharply and since the mid 90s has been converging to West levels. Figure 5 depicts the development of the TFR in both, East and West Germany. The 2001 and 2002 values refer to the TFR of unified Germany. The tendency of the East German TFR to approach West German levels becomes apparent. It seems justified to assume that this process is sufficiently advanced to model the TFR based on West German rates only.

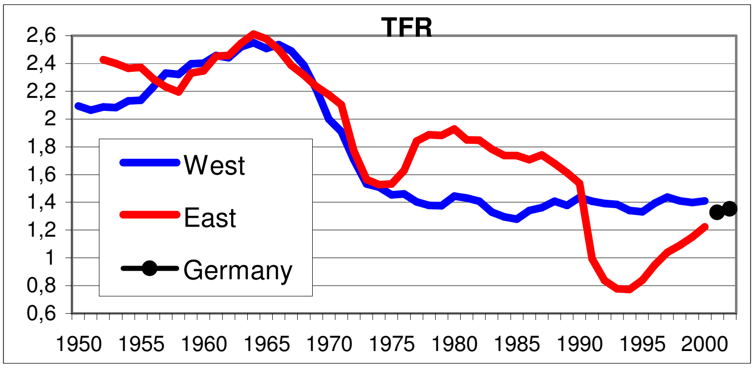

Figure 5: TFR, West and East Germany

It is important to note that convergence of the TFR does not imply convergence of actual fertility behaviour. As shown in Figure 6, convergence of mean and modal age at childbearing is not that advanced yet, though it might eventually be achieved. This is in line with the findings of Kreyenfeld (2003) and Lechner (2001).

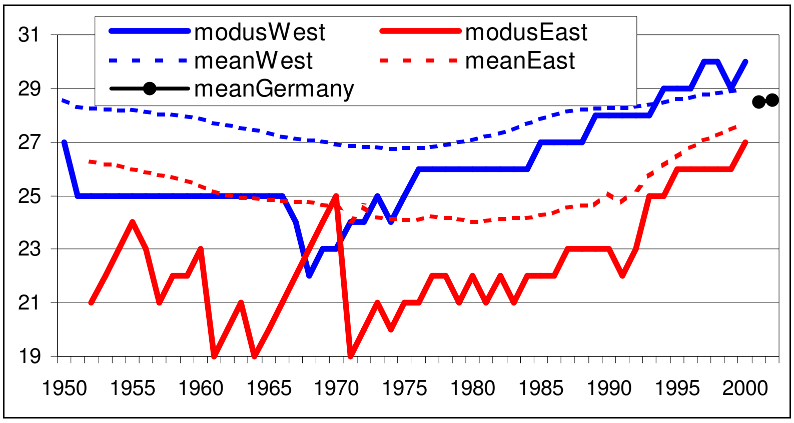

Figure 6: Mean and modal age at childbearing, West and East Germany. Values for 2001 and 2002 are for the entire German population

Lechner (2001, 67) e.g. states, the "East German completed fertility rate for the different ages is never below West German levels" although the Eastern period TFR is drastically below the West. Kreyenfeld (2003, 305) emphasises that "period fertility indicators are easily misinterpreted, particularly when there are changes in the timing of childbirth." With the help of the analysis of cohort fertility, she showed that the Eastern cohorts are still faster in having their first child, although an adaptation has already taken place. Equally, first birth risks are still above West German levels. However, East German women have a lower transition rate to the second child (320). As a conclusion, there is no evidence to assume a "general and rapid convergence of *behaviour* in the old and new federal states" (324).

However, in the context of population forecasting the total fertility rate is more important than mean and modal age at childbirth or parity-specific considerations. It is more important to correctly project the number of births than their timing, even though there are repercussion effects from timing to the number of people born. Given that the total fertility rate in the East has approached West levels, we conclude that modelling fertility based on West German rates only is a valid approximation.

4. Mortality

4.1 Historical Patterns

The development of the age specific mortality rates is characterised by a continuous decline, resulting in a constantly increasing life expectancy, with life expectancy in the West being higher than in the East since the mid 70s. Around German reunification life expectancy in the East was approximately three years lower than in the West, but since then there has been a rapid catching up. Hence, also with respect to mortality the German reunification can be interpreted as a structural break in the Eastern time series. Figure 7 shows the development of life expectancy at birth, separately for sex and region. In terms of life expectancy, the adaptation process from East to West Germany seems to be almost complete.

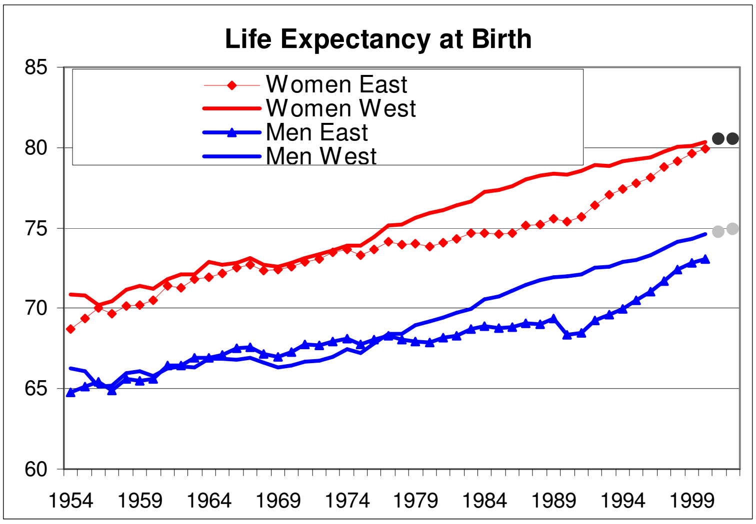

Figure 7: Development of life expectancy at birth, men and women, West- and East Germany, Data for 2001 and 2002 are for the entire German population

4.2 The Lee-Carter Model

We model and forecast mortality with a method developed by Lee and Carter (1992). The Lee-Carter (LC) model decomposes the age and time depending mortality "surface":

$$\ln(mort_{x,t}) = a_x + b_x k_t + e_{x,t} \tag{10}$$

with: $mort_{x,t}$: mortality risk at age x during period [t-1,t].
a_x: age specific mean mortality rate, standardised to $a_x=1/T^*\ln(surv_{x,t})$.
b_x: age specific average change of mortality rate (standardised to $\Sigma b_x=1$)
k_t: time series factor („time specific mortality rate")

Essentially, the Lee-Carter model yields the solution by means of the singular value decomposition (SVD), projecting to the first singular value (Pedroza 2002). An empirical analysis of the mortality rates shows that the first singular value (=18.9) is far greater than the second (=3.1)[11]. Therefore, the approximation, i.e. the projection to the vector space spanned by the first singular value is satisfactory.

Recently, the assumption of a constant age specific change of mortality was shown to be too rigid for some countries (e.g. Carter and Prskawetz 2002 for the case of Austria). In Austria, mortality declined more rapidly at higher ages during recent decades, whereas child mortality declined only to a small extend during the last years. We tested this stationarity assumption implicit in the original Lee-Carter model by comparing the differences of observed and LC predicted values of life expectancy (Carter and Prskawetz 2002). We consider the two 20-year intervals from 1954-1973 and 1981-2000, respectively, and confine ourselves to the investigation of males as for them the discrepancies in Carter and Prskawetz are larger than for females.

[11] This holds for West German males, with the time period from 1954-2000 used.

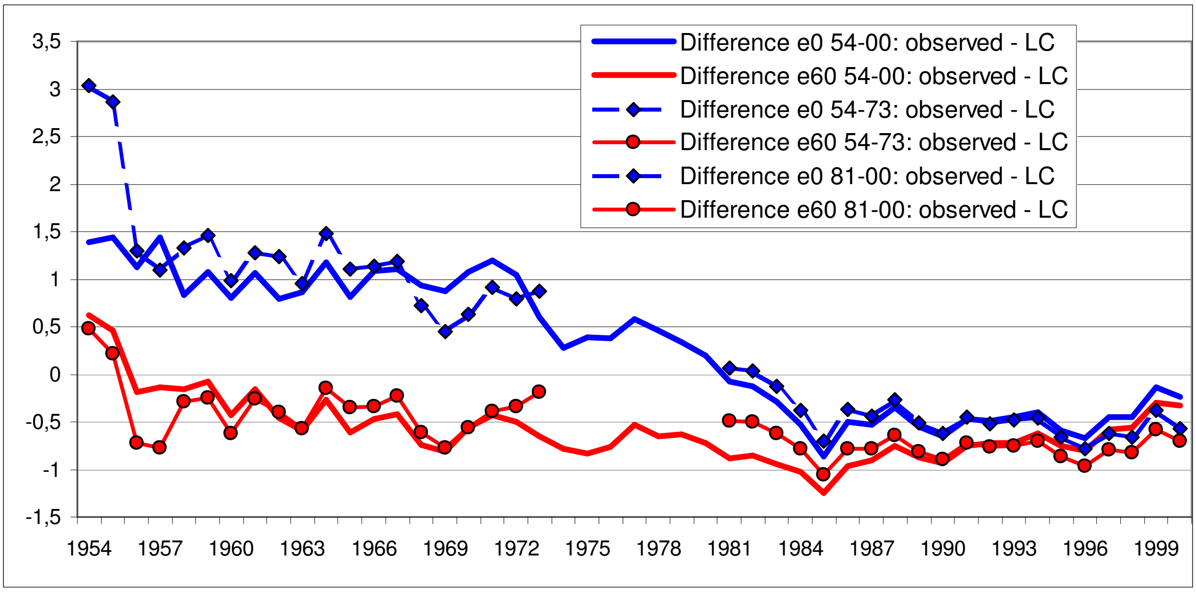

Figure 8: Differences in observed and Lee-Carter estimated life expectancy at 0 and 60 years, males, West Germany, for estimates based on the complete time series 1954-2000 and two selected subsamples

Unlike in the Austrian case, where the LC models which use "their specific" subsamples fit the observed data better than that using the whole sample, the "complete" LC model (bold lines in Figure 8) does an equally good job. Both LC models underestimate e_0 up to the beginning 1970s, and overestimate e_0 afterwards. The estimate is even slightly better for the whole sample. In terms of e_{60}, with the exception of the first two years, e_{60} for the whole time period is overestimated, both for the whole sample and the subsamples. In the case of e_{60}, the whole sample and the two subsamples work equally well[12]. In sum, there are no strong structural shifts; therefore, we use the entire time series in order to estimate the LC model.

After having estimated k_t[13], this series is analysed by a univariate ARIMA model. Neither the autocorrelations nor the partial autocorrelations of the differenced series D.k_t show evidence to add an AR term to the ARIMA(0,1,0) model. The random walk with drift is a common specification. For instance, Tuljapurkar, Li and Boe (2000) also use a random walk with drift in their forecast of life expectancy in the G7 countries, including Germany

Unlike in Lipps and Betz (2004), we take the high correlation of the error terms in the forecast of k_t for men and women into account[14]. This is reasonable since the mechanisms responsible for mortality, especially the medical progress, apply in general equally to males and females.

4.3 East German Mortality

In this section we show that mortality patterns in the East have converged sufficiently close to those in the West to allow for common modelling, that is to apply West Germany mortality rates to the start-off population of the entire German population.

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[12] Figure shows that male's gain in yearly life expectancy for the oldest age group (85+) was (comparably) even higher during 1954-1975, than during 1976-2000, in which the age group 60-85 benefited (comparably) more.

[13] Actually we model k_t as the logit of the survival probability and not the logarithm of the mortality rate. Consequently, the k_t curve increases.

[14] The correlation coefficient of the residual time series of k_t for men and women is .91.

First, the time dependent k_t of women are compared, together with the linear regression fit:

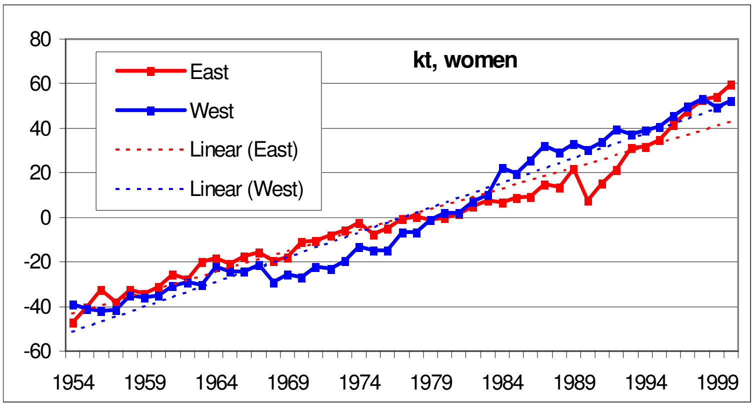
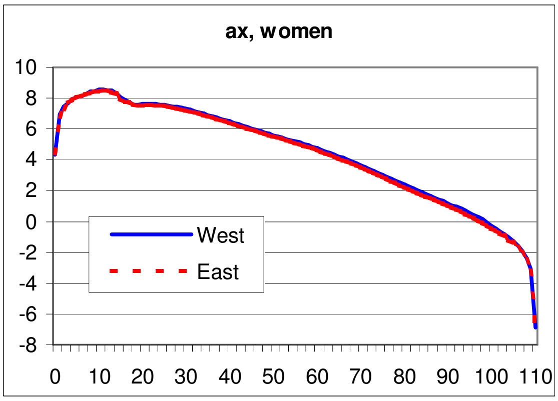


Figure 9: Movement of the k_t of the Lee-Carter model, women, West- and East Germany

In spite of an overall smaller slope of the time specific mortality rate of East German women, a strong adaptation to the western rate can be stated since reunification. Therefore, it seems to be appropriate to model future mortality for the entire German population with the parameters k_t found for the West. Also, the time invariant parameters a_x and b_x exhibit for women very similar profiles for both East and West, as shown in Figure 10.



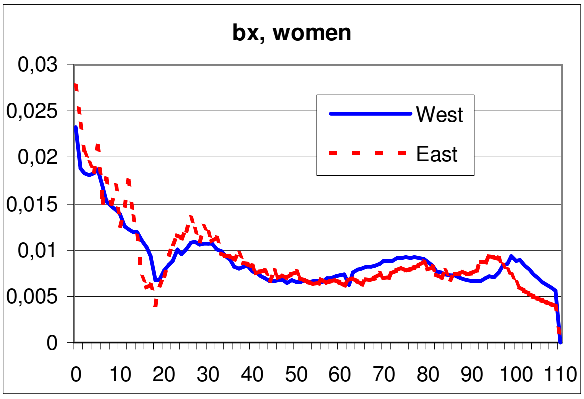


Figure 10: Age specific movement of a_x and b_x in Lee-Carter model, women, West- and East Germany

Considering life expectancy at birth for men, the situation in the East looks slightly different from that in the West. Although a fast catching-up process of the Eastern to the Western figures has taken place since reunification, in 2000 there was still a considerable difference of 1.6 years in life expectancy at birth. Figure 11 shows the development of life expectancy for East German men and also the Lee-Carter recovered curve. The LC model shows a good performance until reunification, but it cannot mirror the fast adaptation process thereafter. However, by 2003, convergence can be assumed to be complete.

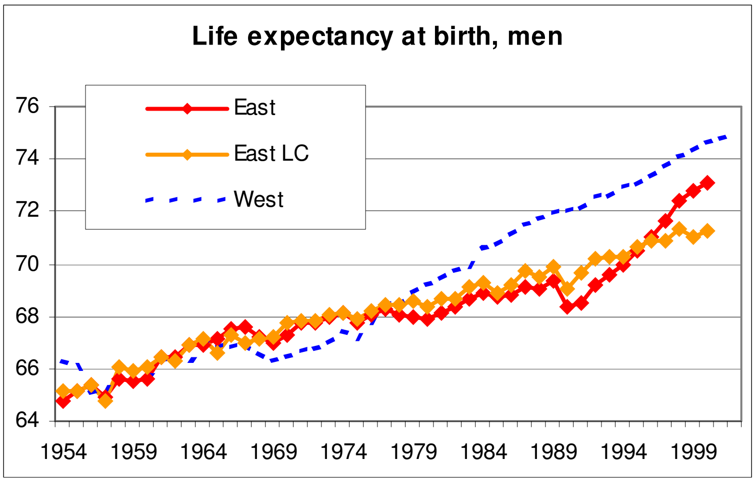
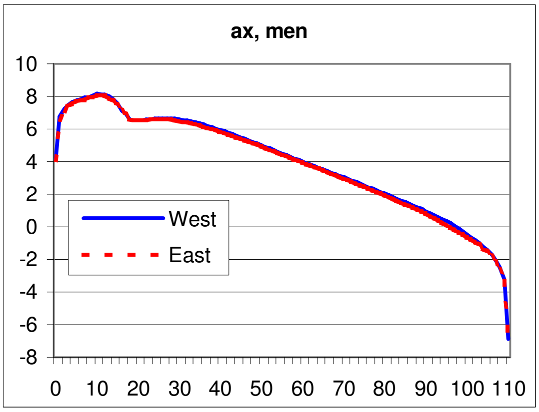


Figure 11: Life expectancy at birth 1954-2000, men, observed development vs. reconstructed LC-model. Data for 2001 and 2002 are for the entire German population

Second, for men, the time invariant parameters are similar in the West and the East. For b_x, there are differences in very young and middle age groups, which nevertheless seem negligible. More interesting is that in East and West different age groups contributed to the gain in male life expectancy in the second half of the 20th century: Whereas for the very young, b_x in the East are above the Western ones, mortality among middle aged men (30-60 years old) has decreased more strongly in the West as shown in Figure 12.



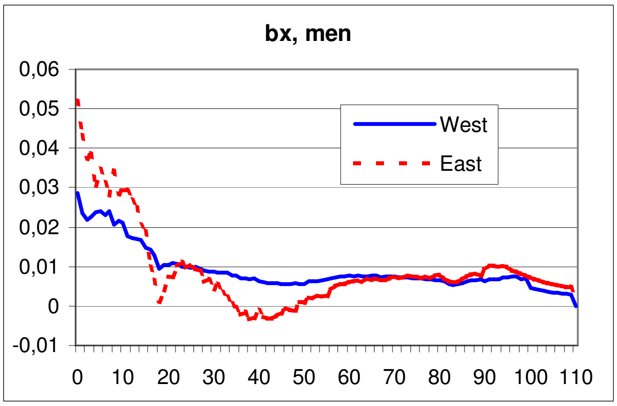


Figure 12: Age specific movement of the a_x and b_x in the Lee-Carter model, men, West - and East Germany

Estimating separate LC-models for the first and the second half of the time period considered, it becomes apparent that in the first half, child mortality decreased faster in the East, with old age mortality decreasing faster in the West. Both differences mostly disappeared in the second half of the period considered, when (small) relative gains in the West were due to decreasing mortality of middle aged men, whereas in the East they were due to the elderly. In sum, it seems reasonable to use the western parameters a_x and b_x in order to project mortality for all German males.

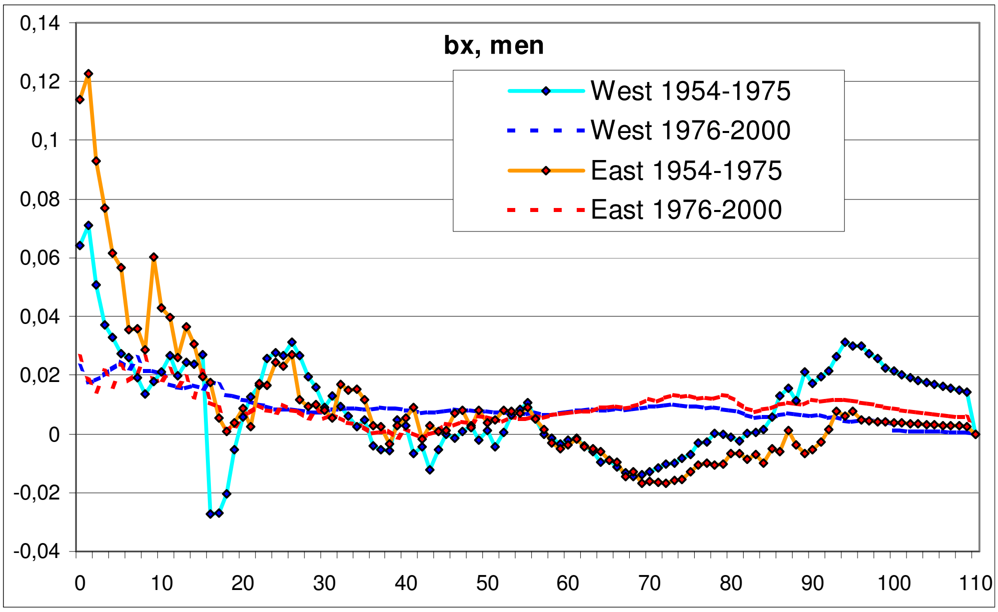


Figure 13: Age specific movement of the b_x in the Lee-Carter model, separately for 1954-1975 and 1976-2000. Men, West - and East Germany

Lee and Carter (1992) finally adjust k_t such that for each year the estimated number of deaths equals the actual number of deaths. In a more recent paper, Lee and Miller (2001) re-estimate k_t so as to reproduce life expectancy. As Booth, Maindonald and Smith (2002) argue, both methods lead to a minimization criterion that is unclear. They adjust k_t by fitting it to the age distribution of deaths. The present study follows the approach by Booth, Maindonald and Smith (2002).

4.4 Results

The stochastic simulation yields 500 trajectories for male life expectancy at birth as displayed in Figure 14:

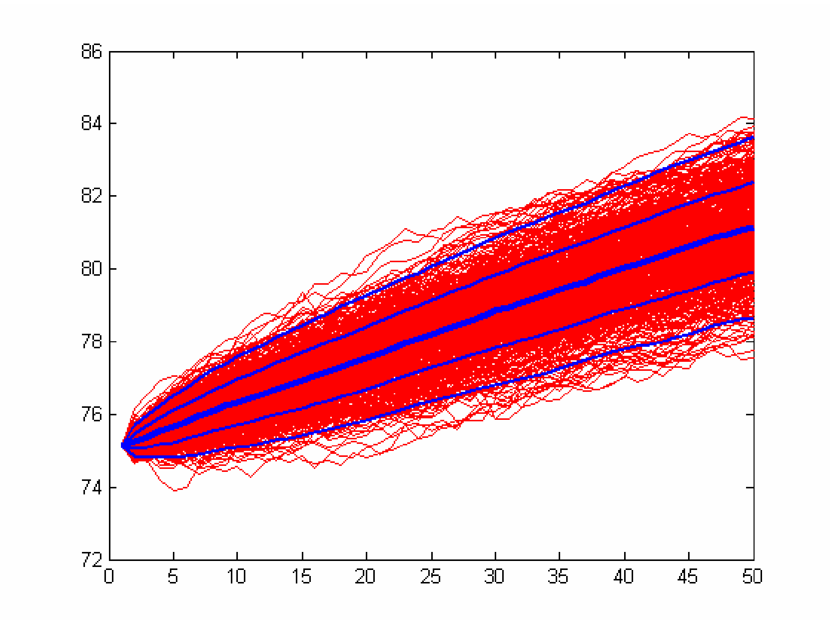


Figure 14: Simulated trajectories of e_0 for males: all (red), and cross sectional mean, +/- one, and +/- two σ-intervals (blue). Forecast horizon 2002-2050. West Germany

The mean life expectancy at birth for males amounts to 81 years in 2050, with a standard deviation of 1.3 years. For women, we simulate a mean of 87 years with a standard deviation of 1.3 years. These figures, together with the official forecasts (Statistisches Bundesamt 2003) are depicted in Table 2.

Source	males	females
Simulation	81 (std=1.3)	87 (std=1.3)
Official forecast: low life expectancy	78.9	85.7
Official forecast: median life expectancy	81.1	86.6
Official forecast: high life expectancy	82.6	88.1

Table 2: Life expectancy at birth in 2050, mean and standard deviation of simulated and officially forecasted figures, Germany

Our simulations are close to the medium official forecasts, but only women's high and low variants are enclosed by the simulated mean +/- one σ-intervals. For males, the extreme variants are within the simulated mean +/- two σ-intervals.

5. Migration

In terms of migration, huge variations have occurred over the last 50 years. On average, 244,000 people per year came to Germany from abroad. This trend is assumed to continue on average, which comes close to the officially forecasted net migration of 200,000 people[15] per year (Statistisches Bundesamt 2003). We assume the current age distribution of the yearly migration vector to remain constant during the forecast horizon, such that the stochastic element is just the net migration.

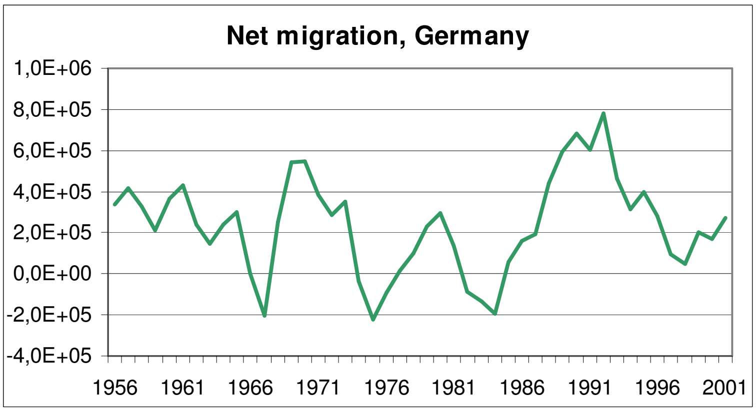


Figure 15: Development of the net migration, Germany, Source: Statistisches Bundesamt (2001)

Standard Box-Jenkins procedures result in an AR(1) process. The estimates are shown in Table 3:

[15] "at least 200,000 but less than 300,000" in the medium variant.

	Coef.	Std. Err.	Z	P>|z|	[95% Conf. Interval]	
const	244453	83300	2.93	0.003	81188	407718
L1.migration	.718	.093	7.71	0.000	.536	.901
σ_ε	158613	18705	8,48	0.000	121952	195273

Table 3: Estimated parameters and goodness of fit of AR(1) model for net migration, Germany

A stochastic simulation of the net migration for Germany results in the following trajectories of the number of net migrants over the course of the forecast horizon, starting from 0 in 2001. The distribution of the number of net migrants in 2050 predicts a mean of around 15 millions, with a standard deviation of about 4.6 millions. In Table 4, these figures are compared to the official projection (Statistisches Bundesamt 2003), which assumes 100.000, 200.000, and 300.000 immigrants per year in the different variants.

Source	Million persons
Simulation	14.96 (std=4.77)
Official forecast: low migration	5.66 (cumulated)
Official forecast: median migration	10.46 (cumulated)
Official forecast: high migration	14.46 (cumulated)

Table 4: Net migrants until 2050, mean and standard deviation of simulated and officially forecasted figures, Germany

When interpreting the figures, it is important to keep in mind that the official numbers are cumulated net migrations, i.e. disregarding mortality and fertility.

6. Results of the Stochastic Population Projection

The simulation is based on the start-off population in 2002. For 2050, 500 stochastic simulations yield a mean population of 77 million people. The standard deviation of the projection equals 7.5 million. We compare the simulated population to the official forecast. Statistics Germany forecasts a population of 67.0 millions in the variant „minimum population“ in 2050, 75,1 millions in the variant „medium population“, and 81.3 millions in the variant „maximum population“ (Statistisches Bundesamt 2003). With a mean of 75,8 million people our forecast is just slightly above the „medium population” variant. Figure 16 shows the simulated trajectories for the total population and the prediction intervals. Simulating the population, by sex and age, the stochastic population pyramid in Figure 17 results for the year 2050.

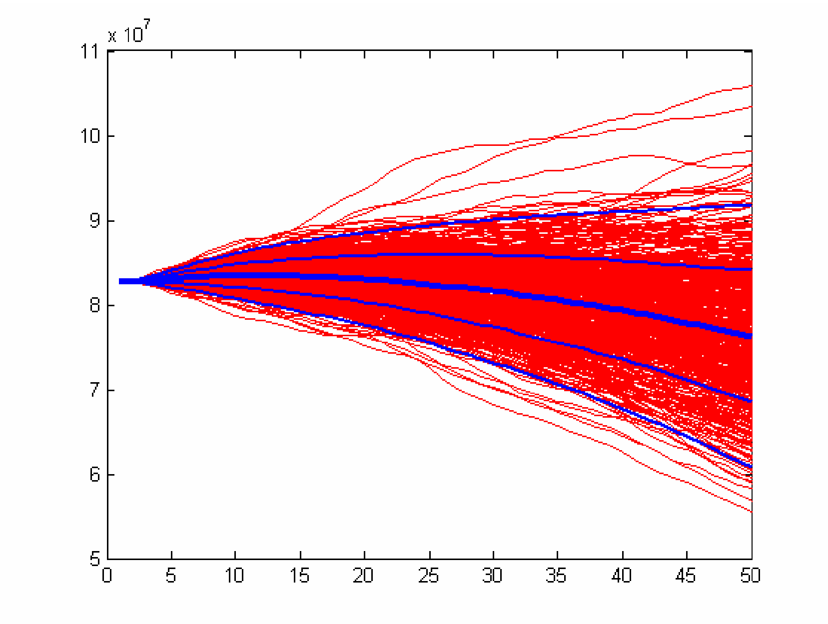


Figure 16: Simulated trajectories of the population number: all (red), and cross sectional mean, +/- one, and +/- two σ-intervals (blue). Forecast horizon 2002-2050, Germany

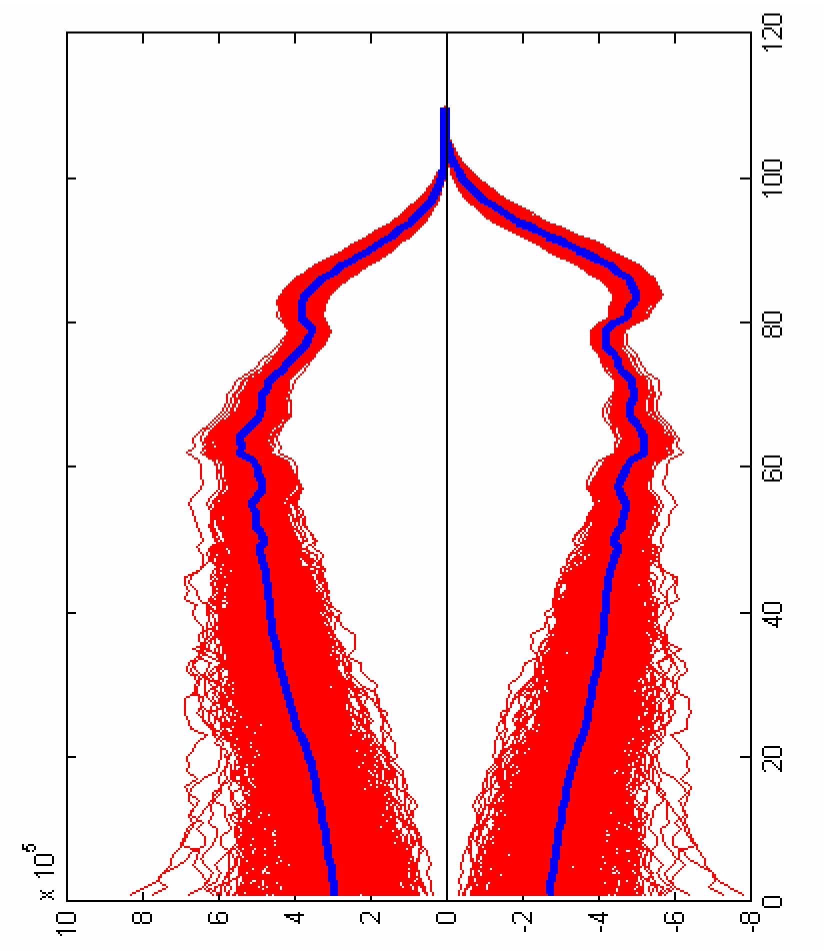


Figure 17: Stochastic population pyramid in 2050 (red), mean (blue). Germany

A prominent application of population forecasts is the calculation of future dependency ratios, in order to assess the future financial burden of the potentially employed population. We define the „Total dependency ratio" (TDR) as the ratio of the potentially employed (approximated by the 20-59 year old population) to the „dependent" population (under the age of 20 or over 59). The course of the simulated TDR is depicted in Figure 18.

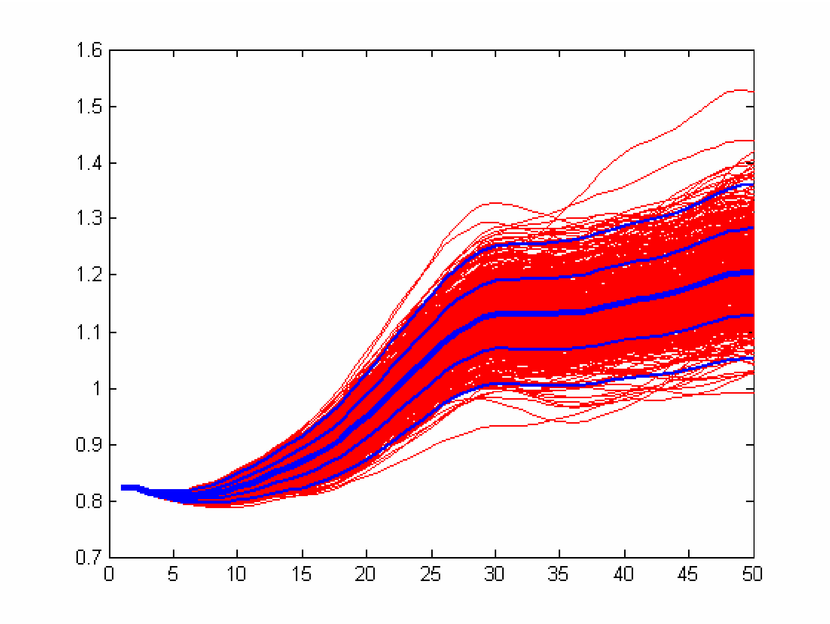


Figure 18: Simulated trajectories of the TDR: all (red), and cross sectional mean, +/- one, and +/- two σ-intervals (blue). Forecast horizon 2002-2050. Germany

The mean TDR in Germany is simulated to be around 1.2 in 2050, with a standard deviation of 0.07. This implies a σ-prediction interval of [1.13; 1.27]. Statistics Germany forecasts the following (Table 5) TDRs, under the assumptions of different life expectancy and migration scenarios (Statistisches Bundesamt 2003). Statistics Germany assumes the TFR to be 1.4 throughout the forecast period.

Variant	Assumption life expectancy	Assumption net migration	Total dependency ratio (0-19 plus 60+ / 20-59 year old pop), in 2050
2	low	high (> 300 000)	1.046
5	medium	medium (~ 200 000)	1,120
8	high	low (< 100 000)	1.219

Table 5: Scenarios and forecasts for TDR in 2050 by Statistics Germany (Statistisches Bundesamt 2003)

Our simulated dependency ratio is higher than the official "medium" variant, however, all scenarios are inside the 2σ-prediction interval of our stochastic forecast.

7. Conclusion

The paper applies and adapts stochastic forecast techniques, in order to model the West and East German population jointly. It is shown, that East German population rates have converged to their western counterparts to a considerable extent, which allows using only the West German historic rates.

The simulation yields a mean population of 77 million people in 2050. This is 1.9 million above the medium variant of the official projection. Regarding forecast uncertainty, the difference between the official maximum and minimum forecast amounts to 14.3 million people, while the difference between the upper and lower bounds of our σ –interval equals 14.8 million people. Similar results hold for the total dependency ratio. We simulate a mean TDR of 1.2 for 2050, which is 0.08 higher than the official figure. The range enclosed by the respective minimum and maximum scenarios equals 0.17. This implies that substantial probability mass is outside the official prediction interval. Thus, compared to our projection the official forecast understates forecast uncertainty.

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