A Technique for Estimating Life Expectancies at Older Ages in Destabilized Populations with Application to Some Developed and Less Developed Countries

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[Extended Abstract]

INTRODUCTION:

Among the methodological investigations carried out in estimating life expectancies at older ages beyond age 70, the techniques developed by Horiuchi and Coale (1980), Mitra (1984) and Lahiri (1990) are worth noting. It may be noted that all the techniques referred above are based on the assumption of sectional stability at older ages (age 65 and above). Because of considerable increase in public health awareness and remarkable development in medical sciences during the later half of the twentieth century, particularly during the last three decades, the overall longevity including that at older ages has also increased considerably worldwide including in many less developed countries. As a result age-specific growth at older ages have also increased considerably over time in various developed and less developed countries, hence the assumption of approximate stability may not be tenable in such countries. Thus, an attempt has been made in this paper to develop a technique for estimating life expectancies at older ages in a destable population following **Generalized Population Model (GPM)** of age-structure applicable to any population (Bennett and Horiuchi, 1980; and Preston, et.al., 1982).

METHODOLOGY:

The Generalized Population Model

According to a destabilized or generalized population model which is applicable to any population, the function **N(x; t)** describing the age-structure of any population at time **t** is given by the following equation (Bennett and Horiuchi, 1981; Preston and Coale, 1982):

$$N(x;t) = B(t)\exp\left[-\int_0^x r(y;t)\,dy\right]p(x;t) \quad \text{..........(1)}$$

where, **B(t)** : Number of births at time **t**;

r(y; t): Instantaneous rate of growth of persons aged **y** at time t;

p(x;t): Probability of surviving from birth to exact age **x** according to the stationary population associated with the destabilized population at time t.

Estimation of the Ratio – e(x+10)/e(x) under GPM using the Age-data at any Two points of Time (not necessarily multiple of 5 or 10 years apart)

In life table terminology **p(x;t) = l(x; t)/l(0; t)**, where **l(x; t)** denotes the number of survivors at exact age **x** out of the initial birth cohort **l(0; t)** in the stationary population.
Taking **l(0; t) = B(t)**, one can easily find the following expression for l(x; t) from the equation (1), for the convenience and simplicity the 'argument' **t** will be omitted henceforth :

$$l(y) = N(y) * \exp\left[\int_0^y r(u)du\right] \quad \text{................ (2)}$$

Now, by definition **T(y)**, the person-years lived beyond age **y**, can be written as:

$$T(y) = N(y+) * \exp\left[\int_0^{C_{y+}} r(u)du\right] \quad \text{............(3),}$$

where **N(y+)** represents the number of persons aged **y** and above. The equation (3) can be obtained by integrating both sides of (2) in the age-range (y, ω), where **ω** being the maximum age attainable by a person in the population under study, and according to the first mean value theorem of integral calculus, there exists a point (age) C_{y+} lying between the ages **y** and **ω** such that the identify (3) holds true. Remembering that **e(x) = T(x)/l(x),** it can be shown by using the equations (2) and (3) for **y = x & x+10** that the ratio **e(x+10)/e(x)** can be expressed through the following formula:

$$\frac{e(x+10)}{e(x)} = \exp\left[\int_{C_{x+}}^{C_{(x+10)+}} r(u)du - \int_x^{x+10} r(u)du\right] \times \frac{b_x}{b_{x+10}} \quad \text{.......... (4),}$$

According to the first mean value theorem of integral calculus there exists two points[1] (ages) -- $\mathbf{C}_{x+}$ and $\mathbf{C}_{(x+10)+}$ in the open intervals **(x, ω)** and **(x+10, ω)** such that the identity (3) holds true for y = x & x+10. The quantities $\mathbf{b}_x [= \mathbf{N(x)/N(x+)}]$ and $\mathbf{b}_{x+10} [= \mathbf{N(x+10)/N((x+10+)}]$ in the above equation, which represent the ***birth-day rates*** or the ***rates of arrival of persons*** at ages **'x'** & **'x+10'** respectively. The values of $\mathbf{b}_x$'s have estimated through the adopted by Preston and Lahiri (1991).

Discrete Approximation of the Ratio – e(x+10)/e(x)

To obtain an approximation of the ratio **e(x+10)/e(x) in discrete case**, it is necessary to evaluate the integrals in the R.H.S. of the formula (4). Assuming that the growth curve ($\bar{\mathbf{r}}_x$) follows a second-degree polynomial[2], an approximate discrete version of the expression in the R.H.S of equation (4) can be obtained through the following approximation after evaluating the first integral through the Simpson one-third rule of numerical integration, and evaluating the second integral by splitting the whole domain of integration (x, x+10) into two sub-intervals of equal size - - (x, x+5) and (x+5, x+10) and noting that $_5\mathbf{r}_y = \int_y^{y+5} \mathbf{r(u)du}, \text{ for } \mathbf{y = x \ \& \ x+5}$:

$$\frac{\mathbf{e(x+10)}}{\mathbf{e(x)}} = \exp\left[\frac{\mathbf{h}^*_{x+5}}{3}\left(\bar{\mathbf{r}}(\mathbf{C}_{x+}) + 4 * \bar{\mathbf{r}}_k + \mathbf{r}(\mathbf{C}_{(x+10)+})\right) - 5 * ({}_5\mathbf{r}_x + {}_5\mathbf{r}_{x+5})\right] * \frac{\mathbf{b}_x}{\mathbf{b}_{x+10}} \quad \text{...(4.1)}$$

or

$$\frac{\mathbf{e(x+10)}}{\mathbf{e(x)}} = \exp\left[\frac{\mathbf{h}^*_{x+5}}{3}\left(\mathbf{r}_{x+} + 4 * \bar{\mathbf{r}}_k + \mathbf{r}_{(x+10)+}\right) - 5 * ({}_5\mathbf{r}_x + {}_5\mathbf{r}_{x+5})\right] * \frac{\mathbf{b}_x}{\mathbf{b}_{x+10}} \quad \text{......(5)}$$

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[1] Though the exact values of $\mathbf{C}_{x+}$ and $\mathbf{C}_{(x+10)+}$ are not known, it can be shown that the two points (ages), mentioned above, are sufficiently close to the mean ages of persons aged 'x & above', and 'x+10 & above' respectively (Lahiri, 1983).

[2] Empirical investigations with the age-data at two points of time of various countries (developed and developing countries) indicate that the growth curve ($\bar{\mathbf{r}}_x$) at ages 45 and above resembles well to a second-degree polynomial (for details, see Lahiri and Menezes, 2004).

The quantities ${}_5r_x$ **'s** and r_{x+} **'s** in equation (5) denote the exponential growth rates of persons in the age-groups **(x, x+4)** and **'x & above'** respectively, and $\bar{r}_k$ represents the exponential growth rate at exact age **k**, the mid-point of the interval **(**C_{x+}, $C_{(x+10)+}$**)**, which can be estimated as a weighted average of either $\hat{r}_{x+}$ **and** $\hat{r}_{(x+5)+}$ **or** $\hat{r}_{(x+5)+}$ **and** $\hat{r}_{(x+10)+}$ depending upon whether the age **k**, the mid-point of the interval **(**C_{x+} , $C_{(x+10)+}$**)**, belongs to the sub-interval $S_1 \equiv (C_{x+}, C_{(x+5)+})$ **or** $S_2 \equiv (C_{(x+5)+}, C_{(x+10)+})$ respectively. The statistic $\hat{r}_{a+}$ (for a= x, x+5, & x+10) stands for the estimated value of the exponential rate of growth of persons ages 'a **& above'** which can be obtained through the following formula:

$$ {}_5\hat{r}_a = \frac{1}{m} . \ln [\, {}_5P_a(z+m) / \, {}_5P_a(z)] \text{ .... (5.1),} $$

The quantities ${}_5P_a(z)$ and ${}_5P_a(z+m)$ **in (5.1)** represent enumerated number of persons in the age-group (a, a+4) at time **z** and z+**m** respectively, **m** being the intercensal interval (not necessarily multiple of 5). The value of $\hat{\bar{r}}_k$ in the formula (5) can be obtained through either of the following approximations (Lahiri 2004):

$$ \hat{\bar{r}}_k = \frac{1}{h_{x+2.5}} [(C_{(x+5)+} - k).\, \hat{r}_{x+} + (k - C_{x+}).\, \hat{r}_{(x+5)+}] \qquad \text{...(5.2)} $$

if k belongs to the sub - interval S_1; or

$$ \hat{\bar{r}}_k = \frac{1}{h_{x+7.5}} [(C_{(x+10)+} - k).\, \hat{r}_{(x+5)+} + (k - C_{(x+5)+}).\, \hat{r}_{(x+10)+}] \qquad \text{...(5.3)} $$

if k belongs to the sub - interval S_2.

The notations $h_{x+2.5}$ and $h_{x+7..5}$, used in (5.2) and (5.3), are the widths of the sub-intervals S_1 **and** S_2 respectively**.** One can easily verify from (5.2) or (5.3) that $\hat{\bar{r}}_k$ will be exactly equal to $\hat{r}_{(x+5)+}$ if the sub-intervals S_1 and S_2 are exactly of equal width, that is, when **k** coincides with $C_{(x+5)+}$, the mean age of persons aged 'x**+5 & above'**.

Now, if somehow one can estimate an **e(x)** value at some older age, say at age **a** where **60 ≤ a ≤ 80,** the other values of **e(x)'s** at older ages beyond age 65 years can be obtained by using the estimated values of the ratio **e(x+10)/e(x)**, denoted by $_5\mathbf{E}_x$, over ages obtained through the formula (5).

It has been shown elsewhere (Lahiri, 1983) that the age-pattern of the ratio $_5\mathbf{E}_x$ remains almost unaltered over various mortality levels ($\mathbf{e}_o^0$) within a specified model mortality pattern, but it differs considerably between different model mortality patterns under the same mortality level ($\mathbf{e}_o^0$). This invariant property of **'E-values'** over a broad range of mortality level within a specified model mortality pattern may be used in identifying a suitable model mortality pattern. After identifying the life table consistent with the observed $_5\mathbf{E}_x$-values from a suitable model life table system, **five sets** of **e(x)** values at ages 60 to 80 for a particular value of **e(x)**, say at age **a** lies between 60 and 80 corresponding the selected life table, may be obtained by using the observed set of $_5\mathbf{E}_x$-values. The average of these **five sets** of **e(x)** values, provides final estimates of e(x) values at older ages.

The Data Used and the Application:

The proposed technique requires enumerated age-data at two points of time, not necessarily multiple of 5 years apart, of a closed population. To test the validity of the procedure, the present technique has been applied to different quality of age-data for various countries starting with Japan (1965-70), followed by Korea (1990-95), China (1982-90) and India (1981-91). The relevant age-data were borrowed from the respective census enumerations of the countries. The estimated e(x) values at older ages, so obtained, are sufficiently close to those obtained through life tables based on age-specific death rates for the above countries during the afore-mentioned periods.

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