Estimating the age at which the 'old' become 'old-old'

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Abstract

The problem discussed is the possible deceleration of the survival curve at older ages, with the age at which this occurs marking the transition from 'old' to 'old-old'. Previous work using the life table aging rate or regression methods indicates that such a deceleration exists for some populations of elderly females but apparently not for males. Results on the rate of change of the survival curve are presented using data from the Australian health and disability surveys of 1988, 1993 and 1998. For this data deceleration does appear to occur for both males and females and also for survival curves in the states of disability-free and disabled.

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ESTIMATING THE AGE AT WHICH THE 'OLD' BECOME 'OLD-OLD'

One way of describing the age at which the 'old' become 'old-old' is that it is the age at which mortality begins to decelerate. It is not clear that such a deceleration occurs but there is strong evidence for its existence in several populations of elderly females. Three methods on deciding whether or not such deceleration occurs and, if so, at which age, will be described. The first is the life table aging rate, which has been used by several authors; the second examines the logistic transform of the age-specific probability of death; and the third estimates the rate of change of the survival curve itself. We also present some results for the disability-free survival curve and survival curves in disabled states.

This paper is methodological and descriptive and no attempt is made to canvass theories as to why there could or should be a deceleration of mortality. For useful discussion we refer to Horiuchi and Coale (1990) and Horiuchi and Wilmoth (1998), which contains an extensive bibliography, and the reliability theory approach of Gavrilov and Gavriola (2001).

THE LIFE TABLE AGING RATE

Horiuchi and Coale (1990), Horiuchi and Wilmoth (1998) and others use the age-specific proportional risk of mortality change with age, called the *life table aging rate*. If $\mu(x)$ is the force of mortality at age x this quantity is

$$k(x) = \frac{1}{\mu(x)} \times \ \{\text{rate of change of } \mu(x)\ \} = \frac{\mu'(x)}{\mu(x)} = \frac{d}{dx}\log \mu(x)\,.$$

In the populations studied they found that $k(x)$ reached a maximum at about age 75 for females but that such a maximum did not appear to exist for males. The implication is that female mortality decelerated from about age 75 for these populations; that is, in our terms, the female 'old' became 'old-old' at about that age but the concept was non-informative on matters of male mortality.

To illustrate what can happen, consider the example of logistic mortality with

$$\log \mu(x) = a + bx\ .$$

Then $k(x) = b$, a constant, and there is no deceleration of mortality. On the other hand, if

$$\log \mu(x) = a + bx + cx^2 + dx^3 ,$$

so that

$$k(x) = b + 2cx + 3dx^2 ,$$

then there may be deceleration at an older age, depending on the sign and magnitude of the coefficients.

Working with the life table aging rate is in many ways equivalent to working with the survival curve

$$p_0(x) = \exp\left\{-\int_0^x \mu(u)du\right\}.$$

Observe that differentiation gives

$$p_0'(x) = -\mu(x)p_0(x) ,$$

and hence

$$\mu(x) = -\frac{p_0'(x)}{p_0(x)} = -\frac{d}{dx}\log\left(p_0(x)\right).$$

Then,

$$k(x) = \frac{d}{dx}\log\left[-\frac{d}{dx}\log\left(p_0(x)\right)\right].$$

This expresses the life table aging rate in terms of the survival function. Our preference is to study the latter directly, as will be done below.

TRANSFORMING THE AGE-SPECIFIC PROBABILITY OF DEATH

Given mortality data by age *x* over a period of years, $t_0 < t < t_1$, that is, data for a rectangle in the Lexis plane, Heathcote and Higgins (2004) first estimated the mortality surface

$$\delta(t,x) = \log\left[\frac{q(t,x)}{1-q(t,x)}\right],$$

where $q(t,x)$ is the age-specific probability of death in the interval $((t,x),(t+1,x+1))$ for a person aged *x* at time *t*.

The main aim of the paper was presentation of a technique of large-sample logistic regression that estimates parameterised $q(t,x)$ by weighted least squares. This is an alternative to the frequency-based methods favoured by actuaries and demographers and is in the spirit of the curve-fitting work of Heligman and Pollard (1980). As an application, Heathcote and Higgins (2004) fitted a polynomial in *t* and *x* to twentieth century Dutch data. Examining the rate of change at a sequence of fixed values of year *t* led to results on the deceleration of mortality by estimating the maximum in age *x* (if any) of

$$k_1(t,x) = \frac{d}{dx}\delta(t,x).$$

For post-war Dutch females the estimated age at which the maximum of $k_1(t,x)$, measured along periods, occurred was observed to increase from about age 70 in 1950 to age about 80 in 1990. That is, if indeed this point marks the transition from 'old' to 'old-old', then there has been an increase of about a decade in this critical age over the forty years from 1950 for this group of females.

No such transition was discernable for Dutch males of the same age over the same period. It is possible that this was due to mortality being dominated by deaths from heart disease, or perhaps Dutch males do not exhibit this characteristic. Also, it was not observed in the pre-war male or female populations. This may be an artefact of the method, or the data, or perhaps particular to The Netherlands, but it is tempting to speculate that the observed deceleration of mortality is a consequence of improved medical and living conditions, and increased longevity, since 1945.

THE SURVIVAL CURVE

Suppose that period data comes in the form of counts of survivors at successive ages. Given a parametric estimate of the survival curve $p_0(x)$ it is possible to examine its derivative $p_0'(x)$ directly, which is a variation on the method of Heathcote and Higgins (2004) noted above. However, we use a different approach as follows:

(a) nonparametrically smooth the observed log-odds of being alive, *i.e.* $\tilde{\xi}_0(y) = \log\{\tilde{p}_0(y)/[1-\tilde{p}_0(y)]\}$, where $\tilde{p}_0(y) = l_0(y)/l_0(0)$ and $l_0(y)$ is the number alive at age *y* (with *y* indexing all the ages at which data is available);

(b) difference the resulting estimates of the log-odds, *i.e.* of $\xi_0(x) = \log\{p_0(x)/[1-p_0(x)]\}$, and thereby obtain estimates of $\xi_0'(x)$;

(c) transform the estimates of log-odds and their derivatives back to the probability scale according to the equations $p_0(x) = [1+\exp\{-\xi_0(x)\}]^{-1}$ and $p_0'(x) = \xi_0'(x) p_0^2(x) \exp\{-\xi_0(x)\} = \xi_0'(x) p_0(x)\{1-p_0(x)\}$.

This method avoids the possibility of probability estimates which are less than 0 or greater than 1, and it is the procedure followed to obtain the results presented in the rest of the paper.

For selected years of Australian data, the ordinary survival curve is considered first and then other survival curves. The data comes from comparable national health and disability surveys carried out by the Australian Bureau of Statistics (ABS) in 1988, 1993 and 1998 (ABS, 1998, 2001). Sample sizes were large - about 20,000 of each sex for each of the three years, with individuals classified by age as disability-free, severely disabled, or other disabled. A parametric analysis of part of the data is in Heathcote *et al*. (2003), with further detail and S-PLUS programs in the working paper Davis *et al.* (2002). The main aim of that work was to obtain a parametric estimate of the survival and disability-free survival curves and health expectancies for persons aged 60 and over. The method presented is a parametric way of dealing with multistate life tables and multivariate grouped discrete data that is different to the classical work reviewed, for example, in Land and Rogers (1982) and Schoen (1998), as well as to the methods based on the logistic transform in Brunsdon and Smith (1998) and Millimet *et al*. (2003).

Figure 1 presents smoothed survival curves $p_0(x)$ and their derivatives $p_0'(x)$ for Australian females in 1988, 1993 and 1998. As described in (a), (b) and (c) above, the approach differs from the parametric analysis mentioned in the previous paragraph and the curves are smoothed versions (by cubic splines) of relative frequencies for

$p_0(x)$ and of smoothed differences for the derivatives in Figure 1(b). Observe that there has been an improvement in the length of life over the decade, and that the derivatives achieve turning points at ages in the early eighties, indicative of the transition to 'old-old'. Figure 2 gives the corresponding plots for males. It is interesting to observe that the deceleration of mortality also exists for this group of males, commencing at ages a few years less than for females. As can be seen from the last column there does appear to be an increase of two or three years over the decade in the age at which deceleration commences but the period is too short for any trend to become apparent.

<Figures 1 and 2 about here>

OTHER SURVIVAL CURVES

To exploit further the 1988, 1993 and 1998 disability data made available by the ABS, let us label the disability-free state as state 1, the severely disabled state as state 2, other disabled as state 3, and death as state 4. Being alive, state 0, is then the composite state encompassing states 1, 2 and 3. Thus

$$p_0(x) = p_1(x) + p_2(x) + p_3(x) = 1 - p_4(x),$$

where

$$p_i(x) = \Pr(\text{An individual is in state } i \text{ at age } x), \quad i = 0,1,2,3,4.$$

As age x varies, the probabilities $p_1(x), p_2(x)$ and $p_3(x)$ trace out respectively the survival curves of the disability-free state, the severely disabled state and of the state of other disabled. They provide a decomposition of the survival curve, and the question that will now be taken up is whether or not these curves exhibit a similar deceleration at older ages.

Smoothing is done *via* the log odds, or log ratios,

$$\xi_i(x) = \log\left[\frac{p_i(x)}{p_4(x)}\right], \quad i = 1,2,3.$$

Here, state 4 (death) is taken as the so-called reference state, and smoothed log odds lead to smoothed probabilities through the formulae:

$$p_4(x) = \left[1 + \sum_{i=1}^{3} e^{\xi_i(x)}\right]^{-1}$$
$$p_i(x) = p_4(x) e^{\xi_i(x)}, \quad i = 1,2,3.$$

Figure 3 plots frequencies and nonparametric fits for male log odds and probabilities in 1998. Similar results were obtained for females and for 1988 and 1993. We argue that fitting cubic splines to the log odds is a simple method that yields an acceptable degree of smoothing. It has the advantage that the log odds can take on unbounded positive and negative values.

It is important to recall that these survival curves will generally not be non-decreasing and that therefore estimates of the derivatives may not be smooth. This is borne out in Figures 4, 5 and 6 which plot estimates of the derivatives obtained for females from the smoothed log odds using the equation

$$p_i'(x) = p_i(x)\xi_i'(x) - p_0'(x)\exp\xi_i(x), \quad i = 1,2,3.$$

<Figures 3,4,5,6 and Table 1 about here>

Observe that estimates of the derivatives of $p_1(x)$ (disability-free) and $p_3(x)$ (other disabled) are particularly erratic and that this is reflected for the former in the interval estimates of Table 1. However, our concluding comment is that it is possible to infer in all Australian cases studied that deceleration does occur, although there is an element of vagueness in the estimates of the age of commencement.

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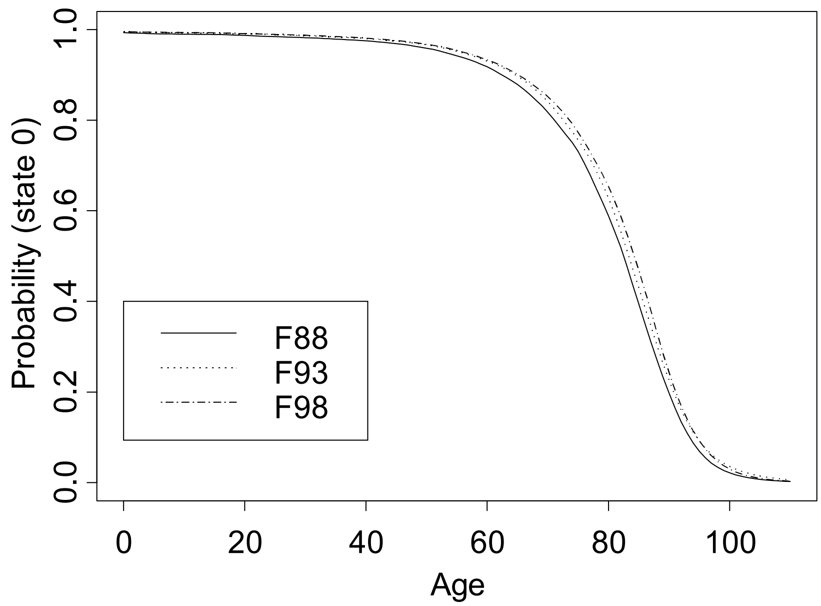
Table 1 Estimates of age at which deceleration commences

		Survival state			
		Ordinary (0)	Disability-free (1)	Severely disabled (2)	Other disabled (3)
1988	F	85	70-81	91	80-85
	M	80	65-80	87	78
1993	F	84	75-82	91	83
	M	84	65-70	89	84
1998	F	88	77-82	92	87
	M	83	74	89	83

Figure 1 Australian females 1988, 1993, 1998: Estimated survival curves (a) and their derivatives (b).



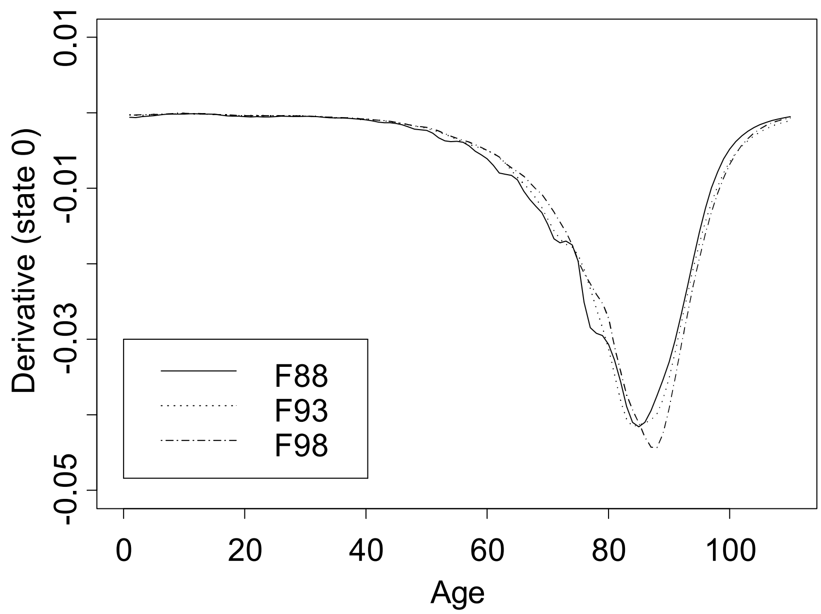
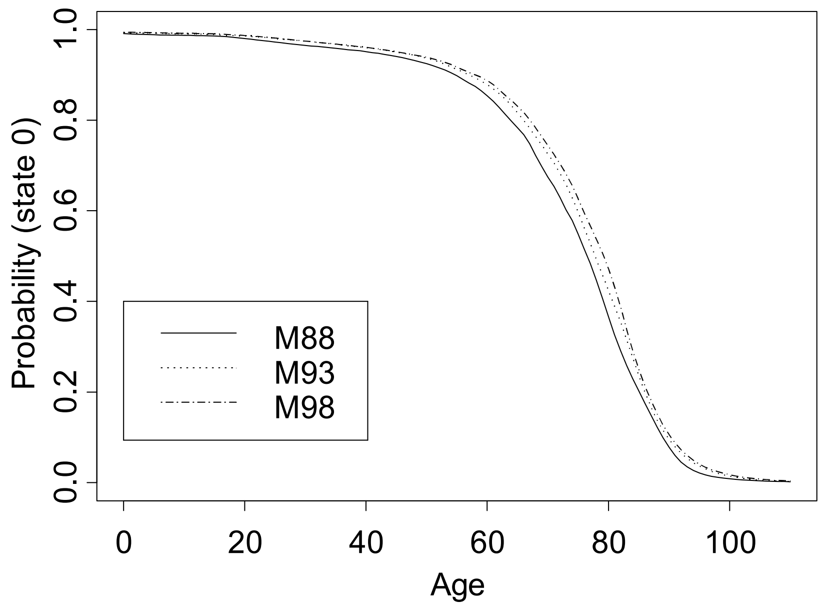


Figure 2 Australian males 1988, 1993, 1998: Estimated survival curves (a) and their derivatives (b).



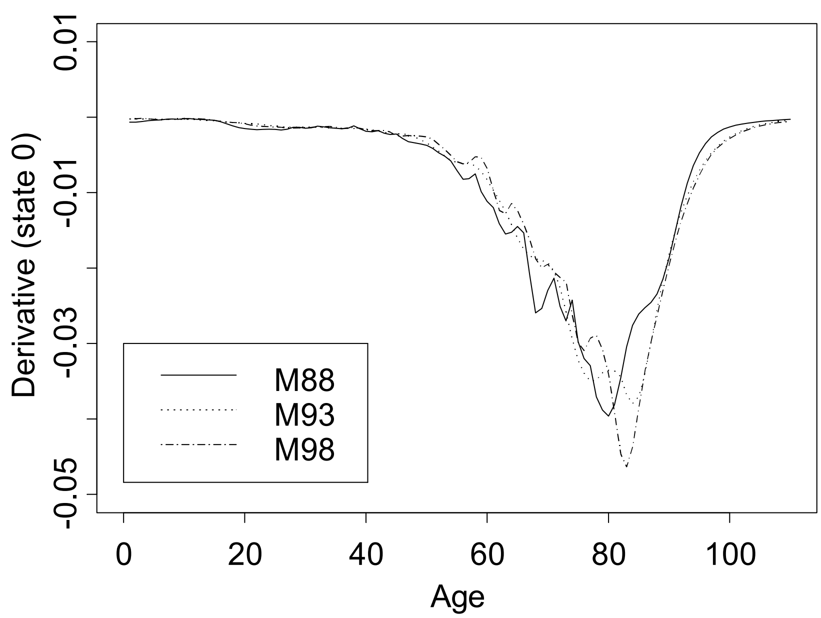
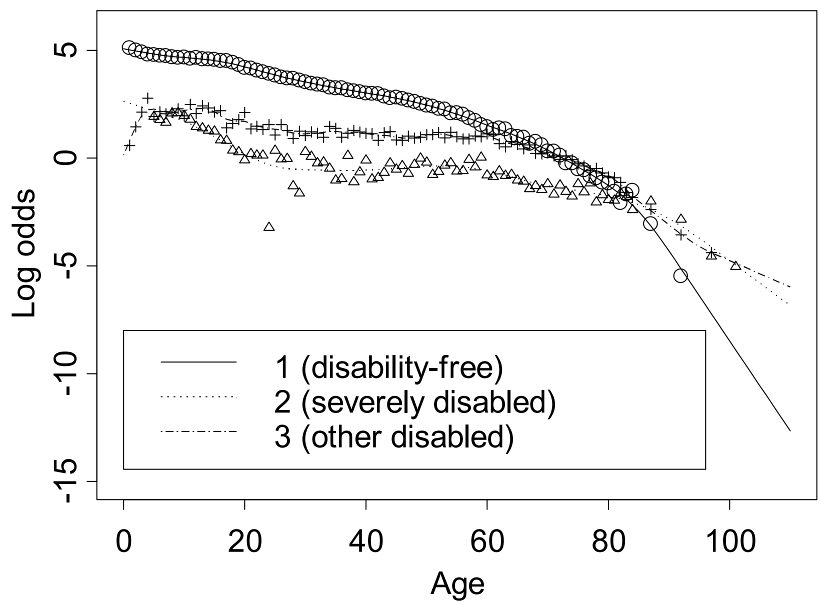


Figure 3 Australian males 1998: Observed and estimated log odds (a) and observed and estimated survival curves (b).



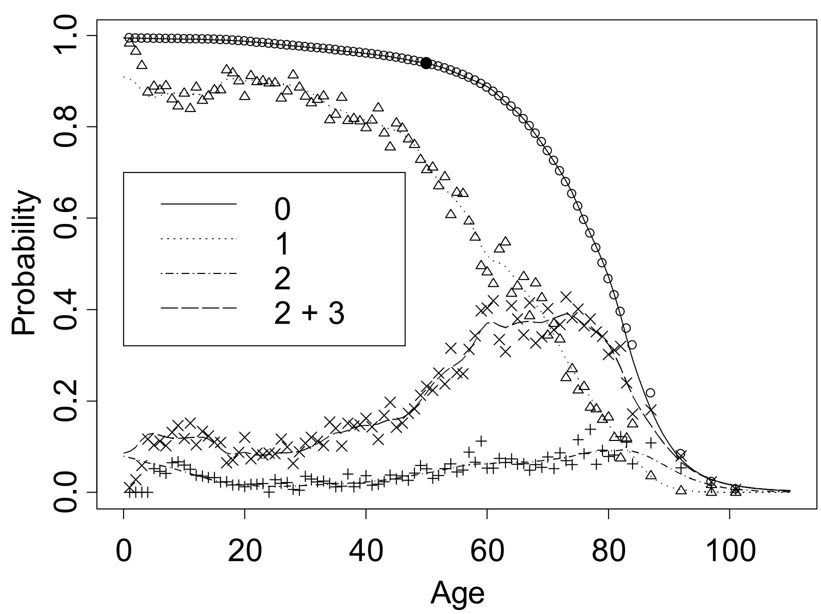
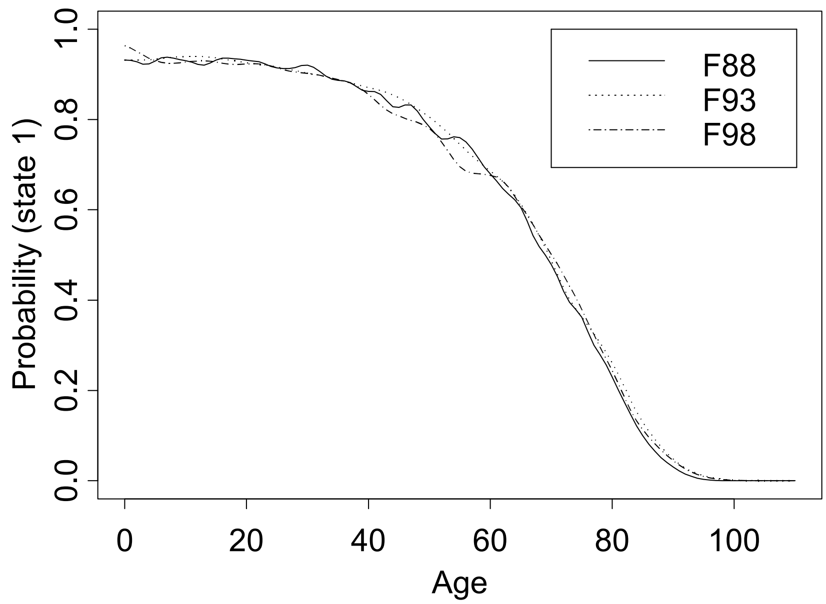


Figure 4 Australian females 1988, 1993, 1998: Estimated disability-free survival curves (a) and their derivatives (b).



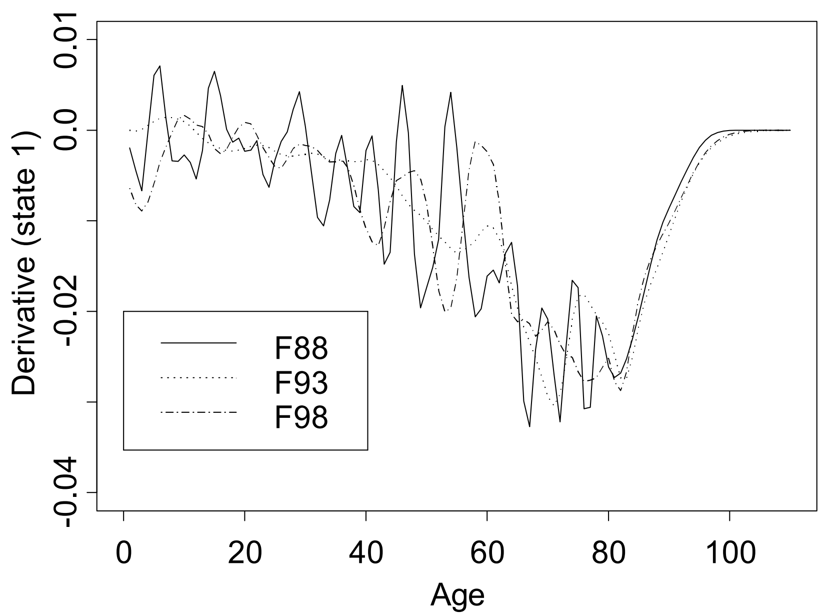


Figure 5 Australian females 1988, 1993, 1998: Estimated severely disabled survival curves (a) and their derivatives (b).

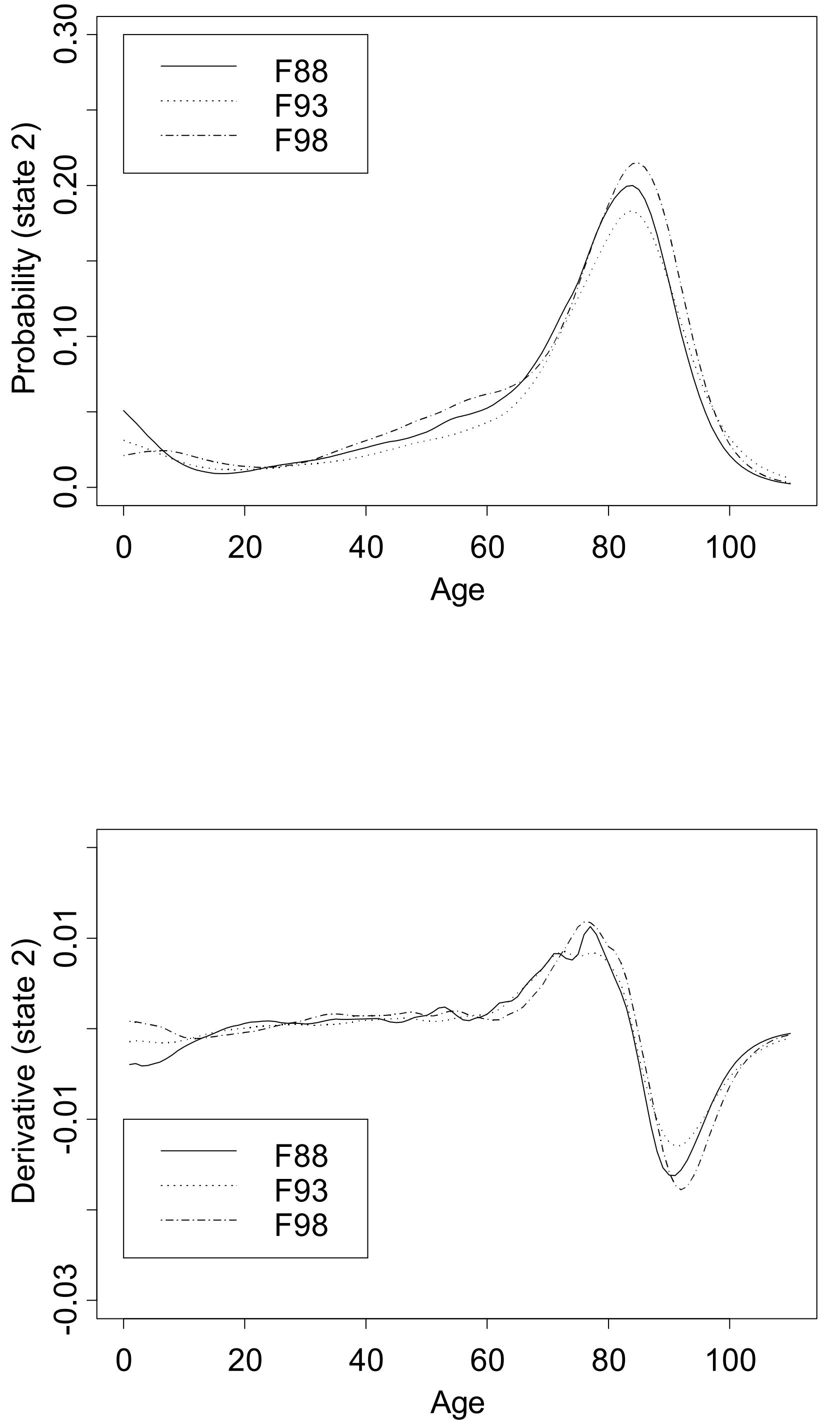


Figure 6 Australian females 1988, 1993, 1998: Estimated other disabled survival curves (a) and their derivatives (b).