Reliability of Intercensal Population Estimates and Now-casts

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Abstract

A stochastic version of the cohort-component method of population forecasts is studied in the paper. Assuming that the evolution of the population can be modelled with a Markov process, where the number of deaths in each age group and births to females of fertile ages follow Poisson distributions it is possible to derive interval estimates for the composition of the population at a given moment of time in the future, if the initial population composition and the parameters of the process – e.g. age specific mortality and birth rates – are given. It is shown that for large enough populations the resulting confidence intervals for forecasted population will be very tight, therefore in the case of short term population forecast of larger countries it is enough to forecast the expected population counts. Results are illustrated using recent Hungarian census, mortality, birth, and migration data.

1. Intercensal population estimates

In the case of intercensal population estimates the task is to determine updated population counts from previous given counts – say census counts – and from vital statistical records. Which is essentially a deterministic problem, though it is possible to enhance results – especially for small areas – using regression methods. Traditional methods of intercensal population estimation have been enhanced using various approaches. One of the most interesting approaches is the rolling census in France, where the census and intercensal population estimation is not so clearly separated.

In some developing countries problems of censuses and intercensal population estimates are often much more complex then in developed countries. Customary indirect estimation methods however resolve these problems. In developed countries mortality and fertility registration are reliable enough for most purposes. The main sources of errors of population estimates in developed countries are census coverage error and problems of the registration of migration.

Census undercount can be estimated from postcensal surveys. The probability of missing a member in every stratum of interest must be small enough for a successful census.

2. Population projections

After the first well known naive, but nonetheless of grate historical impact population projection of Malthus in 1798 had to pass one and a half century until the first systematic population projections of 1945. Malthus based his projection on the concept of population growth rate, without much ado supposing exponential growth. The fundamental idea of population projections is to estimate expected future population counts modelling population growth in a Markov process framework. Recursively determining expected age-specific population counts for successive years (or in five year intervals) by a recursion formula

$$\boldsymbol{P}_{t+1} = \boldsymbol{A}_t \boldsymbol{P}_t + \boldsymbol{u}_t ,$$

where the vector $\boldsymbol{P}_t$ is the population composition (vector of population counts) at time t, $\boldsymbol{A}_t$ is the Leslie matrix constructed from survival probabilities and fertility rates and $\boldsymbol{u}_t$ is the age-specific migration balance. For closed populations, e.g. for the population of the world $\boldsymbol{u}_t = 0$. Of course, it is necessary to have forecasts for $\boldsymbol{A}_t$ and $\boldsymbol{u}_t$. Until the end of the past century these forecasts – scenarios – were subject to quiet obscure expert opinion. It was however clearly felt, that something have to be told about the quality of the resulting population projections. The natural requirement, that past and present population compositions should be exactly reconstructed from an earlier population composition using the real historical fertility, mortality and migration data is easily satisfied. As illustrated by graphs 1-5 taken from the Demographic Yearbook for the year 2002. This requirement for stochastic population projection should be satisfied conditioning on the real historical vital statistical rates, which will reduce in fact to the common deterministic population projection.

While census coverage error and unregistered migration are essentially the only causes of possible errors of intercensal population estimates, population projections are subject to considerable errors of various sources related with the stochastic behaviour of the population growth process. A population $\wp_t$ is a set of members changing in time. It is possible to speak about the totality of all time members of the population: $\wp = \bigcup_t \wp_t$. Let $\left\langle \wp_{st} \middle| s \in S \right\rangle$ be a stratified population, where $\wp_t = \bigcup^*_{s \in S} \wp_{st}$. It is supposed that there is an absorbing state $\omega \in S$ corresponding to death. The elements of S are vectors and it is always necessary to stratify by age and sex. Population counts $P_{st} = \left| \wp_{st} \right|$ are of particular interest for us. With a given member p of the population a family of indicator functions $I_p^s(t) = I^{\{p \in \wp_{st}\}}(t)$ is associated. Then we have $P_{st} = \sum_{\wp} I_p^s(t)$. This equation is however simple, but fundamental in the theory of population projections, because it essentially reduces the theory of classical population projections to the problem of projecting populations consisting of a single member.

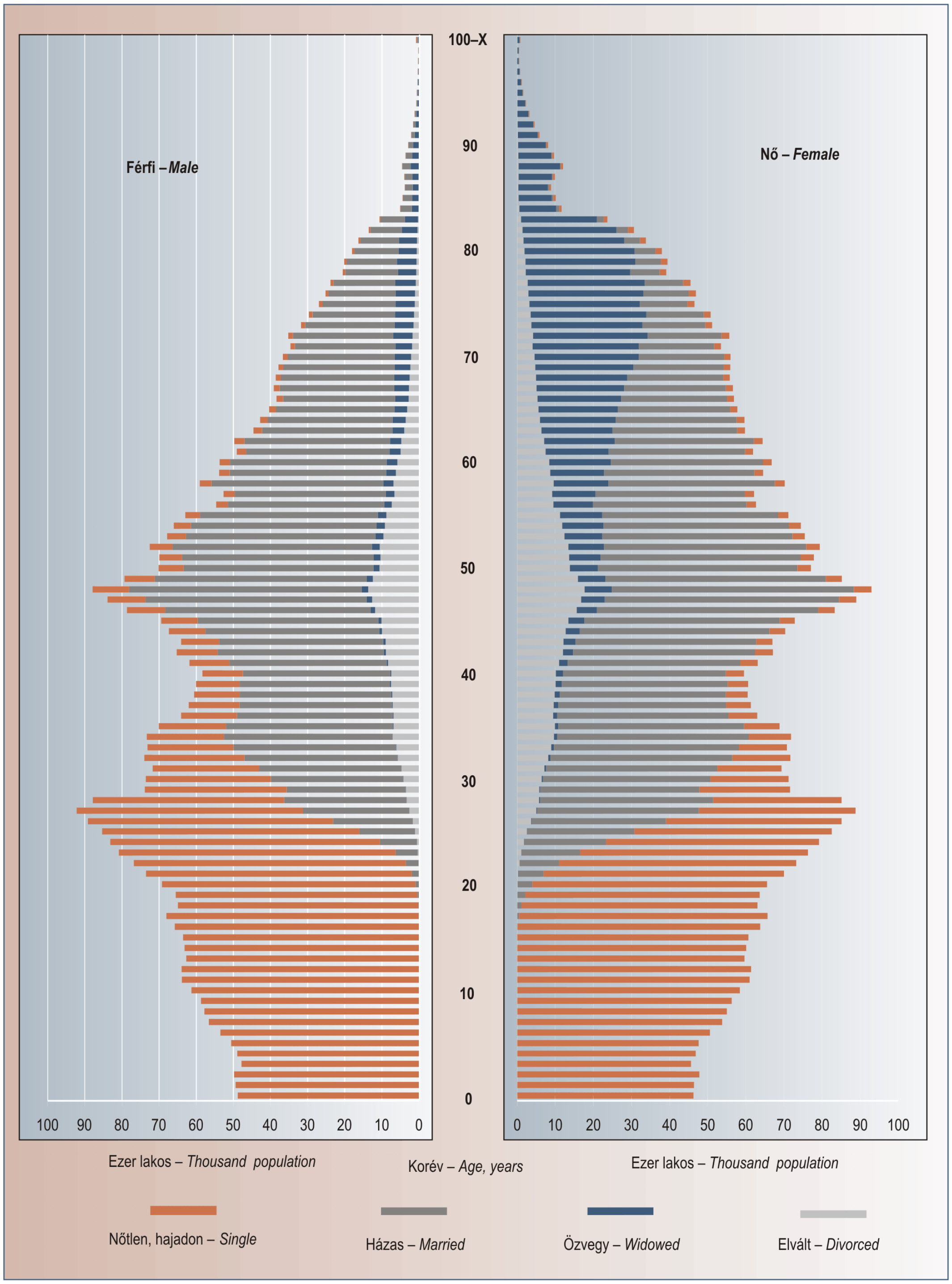
Following revolutionary papers of W. Lutz S. Scherbov and others a nice time series theory have been developed for the forecast of Leslie matrices and the assessment of the error related with the uncertainty of the forecast. AR(1) models seem to be appropriate for this purpose.

It is worth mentioning that even if all transition probabilities are known exactly, that is conditionally on series of given Leslie matrices and given migration there remains uncertainty pertinent to population counts. Really, while knowing transition probabilities and migration no uncertainty remains considering expected population counts, but events of known probability nevertheless remain random. It is easy to see that a population consisting of a single person even knowing exact transition probabilities behaves most unpredictably. However taking into account the model based on Poisson distribution we get that the remaining variance is proportional with *P*, the population size. Therefore the length of confidence intervals will be proportional with $\sqrt{P}$. It turns out that for populations consisting of more than 100,000 people it is possible to get reasonably good predictions.

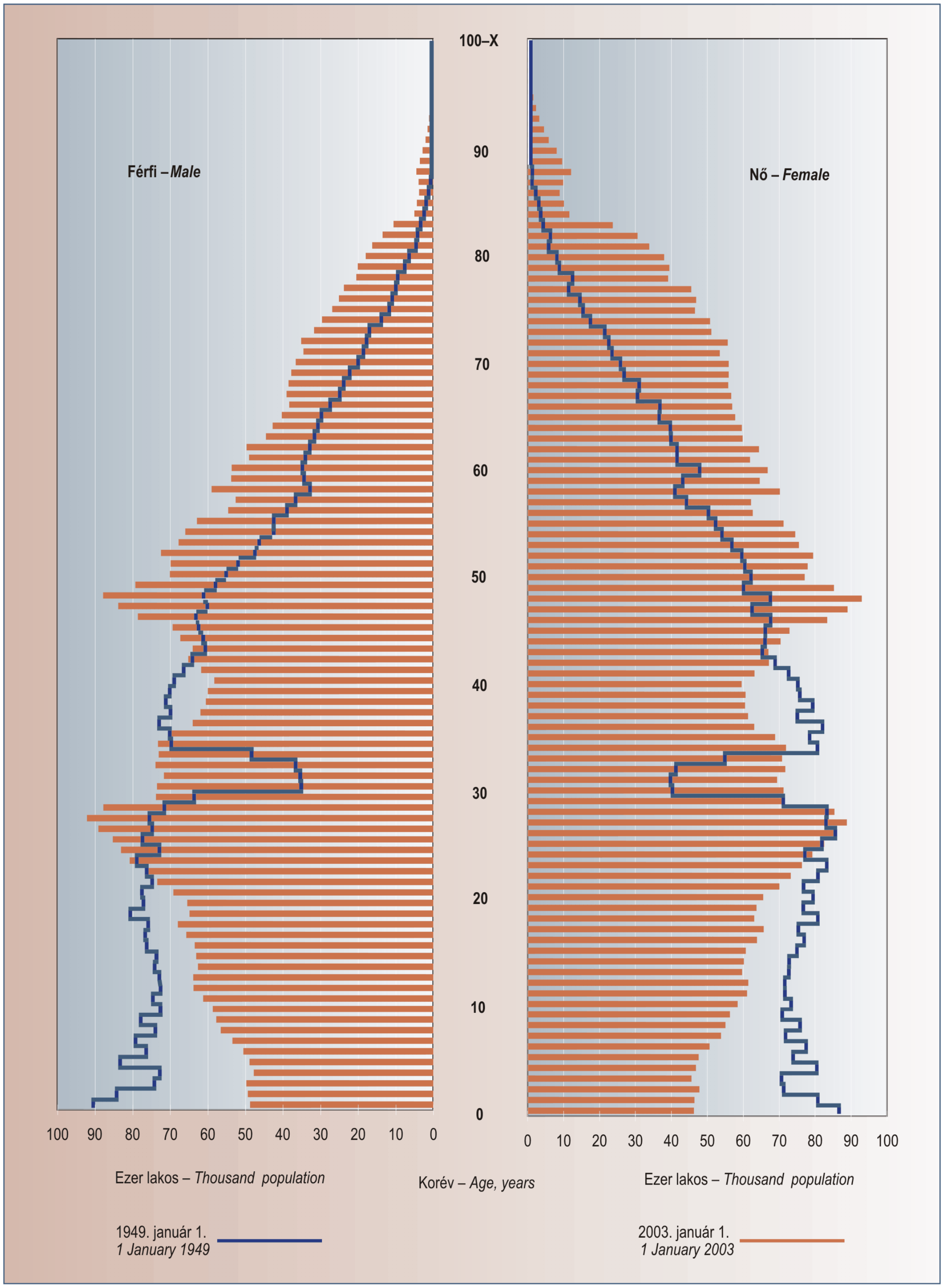
3. Conclusions

As it has been shown, intercensal population estimates and now-casts are generally sufficiently precise estimates. On the other hand long term population projections are subject to growing error mainly arising from the uncertainty related with the realised scenario. Therefore especially financial conclusions derived from long term population projections should be handled cautiously, though for instance there is no doubt about the reality of financial problems of ageing population.

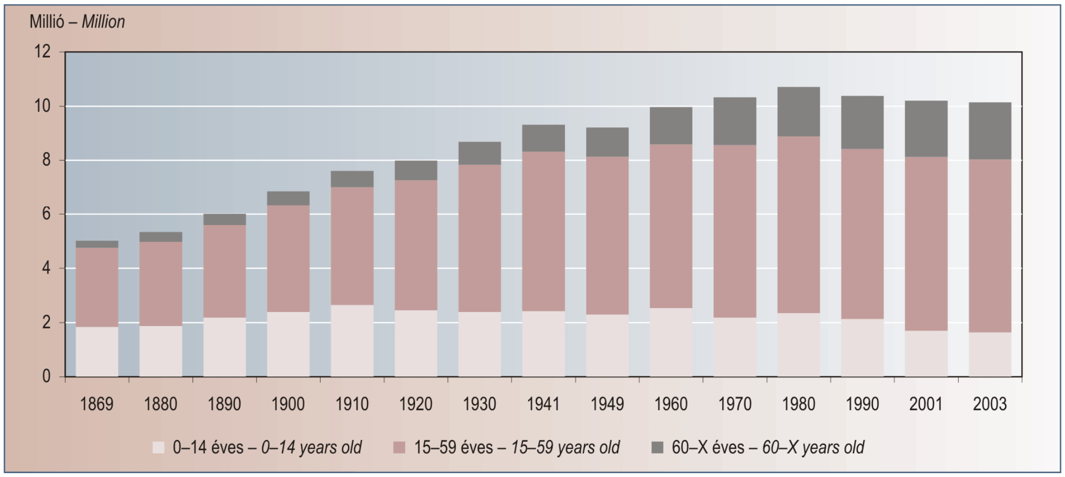
G.1. A NÉPESSÉG SZÁMA NEM, ÉLETKOR ÉS CSALÁDI ÁLLAPOT SZERINT, 2003. január 1.
POPULATION NUMBER BY SEX, AGE AND MARITAL STATUS, 1 January 2003



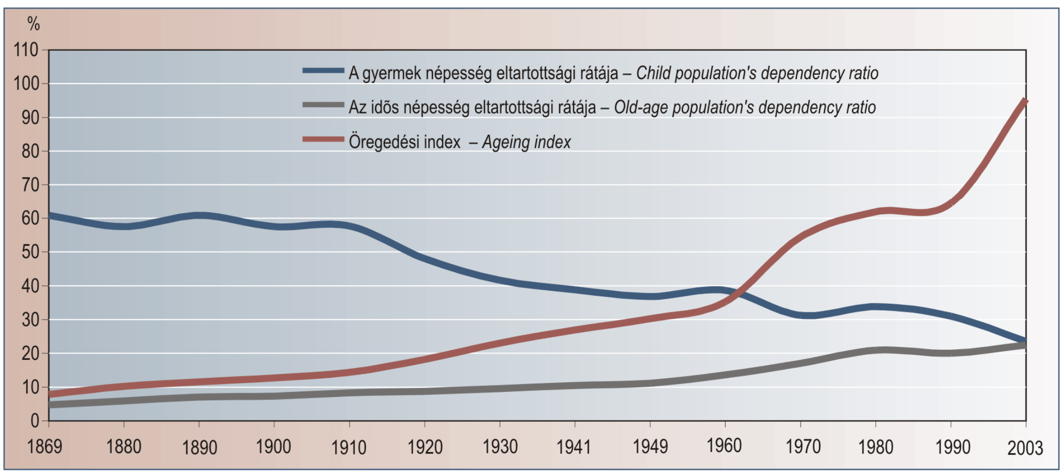
G.2. A NÉPESSÉG SZÁMA NEM ÉS ÉLETKOR SZERINT
POPULATION NUMBER BY SEX AND AGE



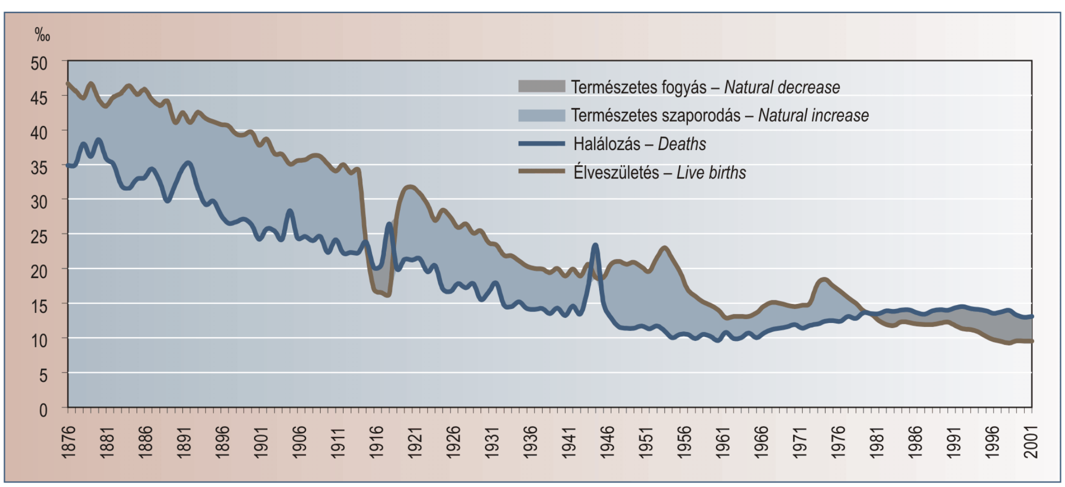
G.3. A NÉPESSÉG SZÁMA FŐBB KORCSOPORTOK SZERINT
POPULATION NUMBER BY MAIN AGE-GROUPS



G.4. ELTARTOTTSÁGI RÁTA, ÖREGEDÉSI INDEX
DEPENDENCY RATIOS, AGEING INDEX



G.5. EZER LAKOSRA JUTÓ ÉLVESZÜLETÉS ÉS HALÁLOZÁS
LIVE BIRTHS AND DEATHS PER THOUSAND POPULATION



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